

WHEN AM I EVER GONNA USE THIS?

Name Key

Period _____ Date _____

You are going to deposit \$800 and let it earn interest until it reaches \$1000, at which point you will use it to buy a totally awesome... uh, mountain bike? (I don't know, something cool.) But you're **smart**, and you want to choose the savings account that will get you to \$1000 the fastest! You have a few choices:

Account A pays 2.75% interest, compounded annually.

Account B pays 2.75% interest, compounded quarterly.

Account C pays 2.75% interest, compounded continuously.

1. First, determine how much money will be in your account after 2 years if you choose... (Hint: Your answers should all be between \$825 and \$850—if you get anything higher or lower, you have made a mistake!)

a. Account A $A = 800 \left(1 + \frac{.0275}{1} \right)^{(1)(2)} = \844.61

b. Account B $A = 800 \left(1 + \frac{.0275}{4} \right)^{(4)(2)} = \845.07

c. Account C $A = 800 e^{.0275(2)} = \$845.23$

2. Now write *equations* to tell you how much you'll have after t years if you choose... (Remember that not every account compounds interest annually!)

a. Account A $A = 800 (1.0275)^t$

b. Account B $A = 800 (1.006875)^{4t}$

c. Account C $A = 800 e^{.0275t}$

3. You are trying to figure out how long it will take you to get \$1000 in the account. Using what we've discussed about solving exponential and logarithmic equations, determine how long it will be before you have \$1000 if you choose...

a. Account A

$$1000 = 800 (1.0275)^t$$

$$1.25 = 1.0275^t$$

$$\log_{1.0275} 1.25 = \log_{1.0275} 1.0275^t$$

$$8.23 = t$$

yrs.

b. Account B

$$1000 = 800 (1.006875)^{4t}$$

$$1.25 = 1.006875^{4t}$$

$$\log_{1.006875} 1.25 = 4t$$

$$32.57 = 4t$$

$$8.14 \text{ yrs.} = t$$

c. Account C

$$1000 = 800 e^{.0275 t}$$

$$1.25 = e^{.0275 t}$$

$$\ln 1.25 = .0275 t$$

$$8.11 = t$$

yrs.

4. Which account should you choose? How long will you have to wait until you can buy that mountain bike?

Account C ; 8.11 yrs.

5. Aspirin has a *half-life* of approximately 3.1 hours. The *half-life* means the amount of time it takes before only half of the substance remains. In other words, if you took 100 mg of aspirin, then 3.1 hours later you would have only 50 mg of it in your system. The following function models the half life of aspirin: $A = A_0(0.5)^{t/3.1}$

a. What percentage of aspirin decays each hour? $(0.5)^{t/3.1}$

- b. A doctor cannot safely operate on a patient with more than 100 mg of aspirin in his system, due to the risk of excess bleeding. (Aspirin is a blood thinner.) If a patient just took a 650-mg dose, how long will it be until the doctor can safely operate?

$$100 = 650(0.5)^{t/3.1}$$

$$0.154 = 0.5^{t/3.1}$$

$$\log_{0.5} 0.154 = \log_{0.5} 0.5^{t/3.1}$$

$$2.7 = \frac{t}{3.1} \quad t = 8.37 \text{ hrs.}$$

6. In 2000, Atlanta's population was 4.2 million people. In 2010, it was 5.2 million. In 2000, Las Vegas' population was 4.8 million people. In 2010, it was 5.6 million.

a. What was the annual rate of growth of Atlanta's population during the period from 2000 to 2010?

$$5.2 = 4.2(1+r)^{10}$$

$$\sqrt[10]{1.24} = \sqrt[10]{(1+r)^{10}} \quad r \approx 0.02$$

$$1.02 = 1+r$$

b. What was the annual rate of growth of Las Vegas' population during the period from 2000 to 2010?

$$5.6 = 4.8(1+r)^{10}$$

$$\sqrt[10]{1.17} = \sqrt[10]{(1+r)^{10}} \quad r = 0.016$$

$$1.016 = 1+r$$

c. Assuming the rate of growth you found for Atlanta in part a continues, write and solve an equation to determine when (in what year) Atlanta's population will be 7.2 million.

$$7.2 = 4.2(1.02)^t$$

$$1.71 = 1.02^t$$

$$\log_{1.02} 1.71 = t$$

$$27.09 = t$$

yrs.

7. You invest \$1000 in an account that pays 4.25% compounded continuously. How much will be in the account after 10 years?

$$A = 1000 e^{.0425(10)} = \$1529.59$$

8. The growth of ants in a colony grows according to the formula $P = 400e^{0.3t}$, where t is measured in weeks. How many ants will be in the colony after 10 weeks?

$$P = 400 e^{0.3(10)} = 8034.21$$

9. You invest \$4,000 in an account that compounds interest continuously. If you want your \$4,000 to turn into \$5,000 after two years, what interest rate do you need?

$$\begin{aligned} 5000 &= 4000 e^{r(2)} \\ 1.25 &= e^{2r} \\ \ln 1.25 &= 2r \\ 0.112 &= r \end{aligned}$$

10. Barometric pressure (measured in pounds per square inch, or psi) decreases as your elevation increases.* For every 1 km increase in altitude, barometric pressure decreases by 11.5%. Today, barometric pressure at sea level is 14.8 psi. At what altitude will the barometric pressure be 13.0 psi?

$$\begin{aligned} 13 &= 14.8 (1 - .115)^x \\ 0.878 &= (.885)^x \\ \log_{.885} 0.878 &= x \\ 1.07 &= x \\ \text{km} \end{aligned}$$

* That's why airplanes have pressurized cabins—otherwise you wouldn't get enough oxygen in the thin air!