Unit 4 Test REVIEW! (Part 1)

Name ________________________________

Period _____ Date ________________

SHOW ALL WORK. NO WORK = NO CREDIT!

Write your answer in standard form. Rationalize denominators where applicable.

Topic 1 Complex Numbers. (Calculators not permitted on this topic.) For extra notes / practice, look in Section 4.4 of the McGraw Hill Algebra 2 textbook.

1. **Extended Response.**

   a. Simplify $i^1, i^2, i^3, i^4, \ldots. i^{12}$. Describe the pattern in the values for the powers of *i*.

   $i, -1, -i, 1, i, -1, -i, 1, i, -1, -i, 1$

   $i^{4n} = 1$ $\quad i^{4n+1} = i$ $\quad i^{4n+2} = -1$

   $i^{4n+3} = -i$, where n is whole

   b. Consider i^6, i^8, and i^{10}.

   a. Evaluate i^6, i^8, and i^{10}. $-1, 1, -1$

   b. *Explain* why an even power of *i* is always a real number.

   By definition, $i^2 = -1$ is a real number. Any even power can be written as i^{2n} where n is whole

2. **Multiply.** Give your answer in standard complex form.

   a. $3i(6 - 5i)$ $\quad -15i^2 + 18i = \underline{15 + 18i}$

   b. $(5 + 7i)(5 - 7i)$ $\quad 25 - 35i + 35i - 49i^2 = 25 + 49 = \underline{74}$

   c. $(-3 + 7i)(1 - 2i)$ $\quad -3 + 6i + 7i - 14i^2 = -3 + 14 + 13i = \underline{11 + 13i}$

   d. $(3 - 2i)^2 = 6 + 12i + 4i^2 = \underline{2 + 12i}$

   e. $(2i)(1 - 4i)(1 + i) = (2i - 8i^2)(1 + i) = (8 + 2i)(1 + i) = 8 + 8i + 2i + 2i^2 = 6 + 10i$

   f. $\sqrt{-12} \cdot \sqrt{-6}$ $\quad -\sqrt{72} = -6\sqrt{2}$

3. **Divide.** Be sure to rationalize denominators.

   a. $\frac{3 - 3i}{4i} \cdot \frac{i}{i} = \frac{3i + 3}{-4} = \frac{3 + 3i}{-4}$ or $-\frac{3 + 3i}{4}$

   b. $\frac{5}{1 + i} \cdot \frac{1 - i}{1 - i} = \frac{5 - 5i}{1 - i^2} = \frac{5 - 5i}{2}$

   c. $\frac{-16 - 3i}{10i} \cdot \frac{i}{i} = \frac{-16i + 3}{-10} = \frac{3 - 16i}{-10}$ or $-\frac{3 - 16i}{10}$

   d. $\frac{\sqrt{-72}}{\sqrt{-12}} = \frac{i\sqrt{72}}{i\sqrt{12}} = \sqrt{6}$

4. Add / Subtract. Simplify each expression as much as possible.

a. $(-2+4i)-(3+9i) = -5-5i$

b. $(5-2i)-2(3+i) = 5-2i-6-2i = -1-4i$

c. $-i+(7-5i)-3(2-3i) = -i-5i+9i+7-6 = 3i+1 = 1+3i$

d. $(5-2i)+(3-2i) = 8-4i$

Topic 2: Solve Quadratic Equations with Complex Solutions For extra notes / practice, look in Section 4.3., 4.5, and 4.6 of the McGraw Hill Algebra 2 textbook.

5. Solve each equation. Solutions may or may not be complex.

a. Solving Quadratic Equations by Taking Square Roots

1. $(x-2)^2 = -36$

$x-2 = \pm 6i$

$x = 2 \pm 6i$

2. $-2(5x+3)^2 = 32$

$(5x+3)^2 = -16$

$5x+3 = \pm 4i$

$5x = -3 \pm 4i$

$x = \frac{-3 \pm 4i}{5}$

3. $4x^2 - 200 = 300$

$4x^2 = 500$

$x^2 = 125$

$x = \pm 5\sqrt{5}$

4. $3(x+4)^2 - 5 = 55$

$3(x+4)^2 = 60$

$(x+4)^2 = 20$

$x+4 = \pm\sqrt{20}$

$x = -4 \pm 2\sqrt{5}$

5. $100 - 2(x+1)^2 = 0$

$(x+1)^2 = 50$

$x = -1 \pm 5\sqrt{2}$

6. $30 - x^2 = 10$

$20 = x^2$

$x = \pm 2\sqrt{5}$

b. Solving Quadratic Equations by Factoring (Hint: Write in standard form)

1. $x^2-x+6=8$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=-1$ or $x=2$

2. $5x^2-4x=12$

$5x^2-4x-12=0$ $\quad 5(-12)=-60$

$5x^2-10x+6x-12=0$ $\quad -10\cdot 6=-60$

$5x(x-2)+6(x-2)=0$ $\quad -10+6=-4$

$(5x+6)(x-2)=0$ $\quad x=-\frac{6}{5}, 2$

3. $8x^2+x-7=0$

$8x^2+8x-7x-7=0$

$8x(x+1)(-7)(x+1)=0$ $\quad x=-1, 7/8$

4. $x^2+14x+13=0$

$(x+13)(x+1)=0$ $\quad x=-13, x=-1$

5. $9x^2-30x+25=0$

$(3x+5)(3x-5)=0$ $\quad x=\pm 5/3$

6. $12x^2-3x=0$

$3x(4x-1)=0$

$x=0 \quad x=1/4$

c. Solving Quadratic Equations by Completing the Square

1. $x^2-6x=-10$

2. $x^2+2x-1=0$

3. $4x^2-8x+1=0$

d. Solving Quadratic Equations by the Quadratic Formula

1. $2x^2+x+3=0$

$b^2-4ac = 1-4(2)(3) = -23$

$x = \frac{-1 \pm \sqrt{-23}}{4}$

$x = \frac{-1 \pm i\sqrt{23}}{4}$

2. $x^2+x-8=0$

$b^2-4ac = 1-4(1)(-8) = 33$

$x = \frac{-1 \pm \sqrt{33}}{2}$

3. $4x^2-8x+1=0$

$b^2-4ac = 64-4(4)(1) = 48$

$x = \frac{8 \pm 4\sqrt{3}}{8}$

$$x^2 - 6x + 9 = -10 + 9$$
$$(x-3)^2 = -1$$
$$x - 3 = \pm i$$
$$x = 3 \pm i$$

$$x^2 + 2x = 1$$
$$x^2 + 2x + 1 = 1 + 1$$
$$(x+1)^2 = 2$$
$$x + 1 = \pm\sqrt{2}$$
$$x = -1 \pm \sqrt{2}$$

$$4x^2 - 8x + 1 = 0$$
$$x^2 - 2x + \frac{1}{4} = 0$$
$$x^2 - 2x = -\frac{1}{4}$$
$$x^2 - 2x + 1 = -\frac{1}{4} + 1$$
$$(x-1)^2 = \frac{3}{4}$$
$$x - 1 = \pm \frac{\sqrt{3}}{2}$$
$$x = 1 \pm \frac{\sqrt{3}}{2}$$

e. Analyze Solutions of a Quadratic Equation with the Discriminant

1. State the discriminant of the quadratic. $5x^2 - 3x - 12 = 0$

$a = 5$ $b = -3$ $c = -12$ $b^2 - 4ac = (-3)^2 - 4(5)(-12) = 9 + 240 = 249$

2. Use the discriminant to determine the type solutions of the equation. $5x^2 - 3x + 1 = 0$

$b^2 - 4ac =$ $(-3)^2 - 4(5)(1) = 9 - 20 = -11$ complex

3. Use the discriminant to determine the number of real solutions of the equation. $9x^2 - 30x + 25 = 0$

$b^2 - 4ac = 0$ one solution

f. Extended Response - Quadratic Equations

a. To complete the square of x^2+bx, a number is added to the expression. *Explain* why the number added cannot be negative.

$x^2 + bx + c$ is a perfect square trinomial
c must be a positive square number
in order to factor into the square of a binomial

b. Consider the equation $x^2-6x=d$. For what values of *d* will the equation have real-number solutions? *Explain.*

$x^2 - 6x - d = 0$ $b^2 - 4ac > 0$ will be real number sol.
$(-6)^2 - 4(1)(c) > 0$ $36 - 4c > 0$ $-4c > -36$
$c < 9$

c. Write two quadratic equations of the form $ax^2 + bx + c = 0$, one of which has real-number solutions and the other having complex-number solutions. Show that your equations have the type of solutions requested.

$9 - 80 = -71$ $a = 2$ $b = 3$ $c = 10$ complex $b^2 - 4ac < 0$ $3^2 - 4(2)(1$
$100 - 24 = 76$ $a = 2$ $b = 10$ $c = 3$ real $b^2 - 4ac > 0$ $10^2 - (4)(2)(3)$

Topic 3: Rational Exponents ... to be continued

1. True or False. For any integer . False

$$a \neq 0,\ a^{1/n} = \frac{1}{a^n}$$

2. Rewrite the expression $7^{1/5}$ using radical notation. $\sqrt[5]{7}$

3. Rewrite the radical expression $\sqrt[4]{64}$ using rational exponents. $64^{1/4}$ or $2^{6/4}$ or $2^{3/2}$

4. Evaluate without a calculator. $81^{3/4}$ $\sqrt[4]{3^4}^{\,3} = 27$

Unit 4 Test REVIEW!

Part 2 Rational Exponents

D 1. Simplify $8^{4/3}$.

a. $\frac{1}{2}$
c. $\frac{32}{3}$
b. 8
d. 16

D 2. Which is equivalent to $81^{-1/4}$?

a. 9 b. 3 c. $\frac{1}{9}$ d. $\frac{1}{3}$

B 3. The volume of a sphere can be given by the formula $V=4.18879r^3$. You have to design a spherical container that will hold a volume of 55 cubic inches. What should the radius of your container be?

a. 13.13 in. b. 2.36 in. c. 3.62 in. d. 2.49 in.

4. Rewrite $7^{1/5}$ using radical notation. $\sqrt[5]{7}$

5. Rewrite $18^{1/6}$ using radical notation. $\sqrt[6]{18}$

True or False:

6. For any integer $a \neq 0$, $a^{1/n} = \frac{1}{a^n}$. False

7. For any integer $n > 0$ and any positive real number a, $a^{1/n} = \sqrt[n]{a}$. True

Evaluate:

8. $81^{3/4}$ 27

9. $16^{5/4}$ 32

10. $16^{-5/4}$ $\frac{1}{32}$

11. The volume of a dodecahedron (a solid with 12 regular pentagons as faces) is $V \approx 7.66312a^3$, where a is the length of an edge. Find the edge length of a dodecahedron whose volume is 1000 cubic centimeters.

5.07 cm

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Simplify:

12. $\dfrac{25^{1/6}}{25^{2/3}}$ $\quad \dfrac{1}{5}$

13. $(5^{4/5} \cdot 5^{4/5})^{-10}$ $\quad \dfrac{1}{5^{16}}$

14. $\dfrac{49^{5/6}}{49^{1/3}}$ $\quad 7$

15. $(7^{3/4})^{-2/9}$ $\quad \dfrac{1}{7^{1/6}}$ or $\quad 7^{\frac{-6}{36}} = \dfrac{1}{7^{\frac{1}{6}}} \cdot \dfrac{7^{\frac{5}{6}}}{7^{\frac{5}{6}}} = \dfrac{\sqrt[6]{7^5}}{7}$

Simplify:

16. $\sqrt[3]{40} \cdot 4\sqrt[3]{5}$ $\quad \sqrt[3]{40} = \sqrt[3]{8} \cdot \sqrt[3]{5} = 2\sqrt[3]{5}$; $\quad 8\sqrt[3]{25}$

17. $\dfrac{\sqrt[3]{2}}{\sqrt[3]{54}} = \quad \dfrac{1}{3}$

18. $\dfrac{\sqrt[5]{9} \cdot \sqrt[5]{81}}{\sqrt[5]{3}} = 3$

Write the expression in simplest form.

19. $\sqrt[3]{\dfrac{1}{5}}$ ANS: $\dfrac{\sqrt[3]{25}}{5}$ $\quad \dfrac{1}{\sqrt[3]{5}} = \dfrac{1}{5^{1/3}} \cdot \dfrac{5^{2/3}}{5^{2/3}} = \dfrac{\sqrt[3]{25}}{5}$

20. $\sqrt[4]{64}$ $\quad 2\sqrt[4]{4}$

21. $\sqrt[3]{54}$ $\quad 3\sqrt[3]{2}$

22. $\sqrt[4]{32}$ $\quad 2\sqrt[4]{2}$

23. $\sqrt[3]{81}$ $\quad 3\sqrt[3]{3}$

Simplify:

24. $x^{1/3} \cdot x^{1/4}$ $\quad x^{\frac{7}{12}}$

25. $\left(\frac{w^{25}}{x^{20}}\right)^{4/5}$ $\quad \frac{w^{20}}{x^{16}}$

26. Simplify $\left(\frac{1}{16}\right)^{1/2}$. $\quad = \frac{1}{4}$

27. What is the value of $125^{-1/3}$? $\quad = \frac{1}{5}$

Simplify the expression. Write your answer using only positive exponents.

28. $(x^{-2/5}y^{1/3})^{15}$ $\quad x^{-10}y^{5} = \frac{y^5}{x^{10}}$

29. $\sqrt[3]{x^6y^{30}}$ $\quad x^2y^{10}$

30. $\frac{25^{3/2} \cdot 25^0}{25^{-5/2}}$ $\quad 5^7$

31. $\sqrt[7]{\frac{x^{14}y^{21}}{z^{-35}}}$ $\quad x^2y^3z^5$

32. The radius r of a circle with area A is given by $r = \left(\frac{A}{\pi}\right)^{1/2}$. What is the radius of a circular pool with an area of 100π ft^2?

10 ft

33. The radius r of a sphere with volume V is given by $r = \left(\frac{3V}{4\pi}\right)^{1/3}$. What is the radius of a medicine ball with a volume of $\frac{500\pi}{3}$ in.^{3}?

5in

34. The side length s of an equilateral triangle can be approximated by the model $s = \frac{2}{3}(3^{3/2}A)^{1/2}$ where A is the area of the triangle. Approximate the side length of an equilateral triangle with an area of 28 square inches.

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