

Simplify to one term.

$$1. \frac{1}{\cot^2 x + 1}$$

$$= \frac{1}{\csc^2 x} = \sin^2 x$$

$$2. \tan^2 \theta (\csc^2 \theta - 1)$$

$$= \tan^2 \theta (\cot^2 \theta)$$

$$= 1$$

$$3. \csc^2 x (1 - \cos^2 x)$$

$$= \csc^2 x (\sin^2 x)$$

$$= 1$$

$$4. \tan\left(\frac{\pi}{2} - x\right) \sec x$$

$$= \cot x \sec x$$

$$= \csc x$$

$$5. \frac{\sin(-x) \cot x}{\sin\left(\frac{\pi}{2} - x\right)} = \frac{-\sin x \cot x}{\cos x} = \frac{-\sin x \left(\frac{\cos x}{\sin x}\right)}{\cos x}$$

$$= -\frac{\cos x}{\cos x} = -1$$

Find all possible solutions in the interval $[0, 2\pi)$.

$$1. 2\cos x + 1 = 0$$

$$\cos x = -1/2$$

$$x = 2\pi/3, 4\pi/3$$

$$2. \sqrt{3} \csc x - 2 = 0$$

$$\csc x = \frac{2}{\sqrt{3}}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$x = \pi/3, 2\pi/3$$

$$3. 11 = 3\csc^2 x + 7$$

$$4/3 = \csc^2 x$$

$$\sqrt{3/4} = \sqrt{\sin^2 x}$$

$$\sqrt{3}/2 = \sin x$$

$$x = \pi/3, 2\pi/3$$

$$4. 1 = \cot^2 x + \csc x$$

$$1 = \csc^2 x - 1 + \csc x$$

$$0 = \csc^2 x + \csc x - 2$$

$$0 = (\csc x + 2)(\csc x - 1)$$

$$\csc x = -2 \quad \csc x = 1$$

$$\sin x = -1/2 \quad \sin x = 1$$

$$x = 7\pi/6, 11\pi/6 \quad x = \pi/2$$

$$5. \tan^2 x - \tan x = 0$$

$$\tan x (\tan x - 1) = 0$$

$$\tan x = 0 \quad \tan x = 1$$

$$x = 0, \pi \quad x = \pi/4, 3\pi/4$$

$$6. \sin^2 \theta = 3\cos^2 \theta$$

$$\sin^2 x = 3(1 - \sin^2 x)$$

$$\sin^2 x = 3 - 3\sin^2 x$$

$$4\sin^2 x = 3$$

$$\sqrt{\sin^2 x} = \sqrt{3/4}$$

$$\sin x = \sqrt{3}/2$$

$$x = \pi/3, 2\pi/3$$

$$7. \sec^2 x - \sec x - 2 = 0$$

$$(\sec x - 2)(\sec x + 1) = 0$$

$$\sec x = 2 \quad \sec x = -1$$

$$\cos x = 1/2 \quad \cos x = -1$$

$$x = \pi/3, 5\pi/3 \quad x = \pi$$

$$8. \sec x \csc x = 2 \csc x$$

$$\sec x \csc x - 2 \csc x = 0$$

$$\csc x (\sec x - 2) = 0$$

$$\csc x = 0 \quad \sec x = 2$$

$$\sin x = \text{und.} \quad \cos x = 1/2$$

$$\text{No solution} \quad x = \pi/3, 5\pi/3$$

Verify the following identities.

$$9. \csc x - \sin x = (\cos x)(\cot x)$$

$$\frac{1}{\sin x} - \frac{\sin x}{1} \left(\frac{\sin x}{\sin x}\right)$$

$$\frac{1 - \sin^2 x}{\sin x}$$

$$\frac{\cos^2 x}{\sin x}$$

$$\frac{\cos x \cos x}{\sin x} = \cos x \cot x \quad \checkmark$$

$$10. \frac{\tan x + \cot y}{(\tan x)(\cot y)} = \tan y + \cot x$$

on separate sheet

$$11. 1 - \cos 2x - 2 \sin x \cos x = 2 \sin^2 x - 2 \sin x \cos x$$

$$1 - (1 - 2 \sin^2 x) - 2 \sin x \cos x$$

$$\cancel{1} + 2 \sin^2 x - 2 \sin x \cos x \checkmark$$

$$12. \cos(\pi + x) = -\cos x$$

$$\cos \pi \cos x - \sin \pi \overset{0}{\sin x}$$

$$= -\cos x \checkmark$$

$$13. \cot^2 x + \sin^2 x + 1 + \cos 2x = \csc^2 x + \cos^2 x$$

$$\cot^2 x + \cancel{\sin^2 x} + 1 + (\cos^2 x - \cancel{\sin^2 x})$$

$$\cot^2 x + 1 + \cos^2 x$$

$$\csc^2 x + \cos^2 x \checkmark$$

$$14. \sin\left(\frac{\pi}{6} + x\right) = \frac{1}{2}(\cos x + \sqrt{3} \sin x)$$

$$= \sin \frac{\pi}{6} \cos x + \sin x \cos \frac{\pi}{6}$$

$$= \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x$$

$$= \frac{1}{2} (\cos x + \sqrt{3} \sin x) \checkmark$$

$$15. \csc(2x) = \frac{\csc x}{2 \cos x}$$

$$= \frac{1}{\sin 2x}$$

$$= \frac{1}{2 \sin x \cos x}$$

$$= \frac{\csc x}{2 \cos x} \checkmark$$

$$16. (\sin x + \cos x)^2 = 1 + \sin(2x)$$

$$= (\sin x + \cos x)(\sin x + \cos x)$$

$$= \sin^2 x + \sin x \cos x + \sin x \cos x + \cos^2 x$$

$$= \sin^2 x + 2 \sin x \cos x + \cos^2 x$$

$$= 1 + \sin 2x \checkmark$$

$$(10) \quad \frac{\tan x + \cot y}{(\tan x)(\cot y)} = \tan y + \cot x$$

$$= \frac{\left(\frac{\sin y}{\sin y}\right) \frac{\sin x}{\cos x} + \frac{\cos y}{\sin y} \left(\frac{\cos x}{\cos x}\right)}{(\tan x)(\cot y)}$$

$$= \frac{\left[\frac{\sin x \sin y + \cos x \cos y}{\cos x \sin y} \right]}{\left[\left(\frac{\sin x}{\cos x}\right) \left(\frac{\cos y}{\sin y}\right) \right]}$$

$$= \frac{\cancel{\cos x} \cancel{\sin y} (\sin x \sin y + \cos x \cos y)}{\cancel{\cos x} \cancel{\sin y} \sin x \cos y}$$

$$= \frac{\cancel{\sin x} \sin y}{\cancel{\sin x} \cos y} + \frac{\cos x \cancel{\cos y}}{\sin x \cancel{\cos y}}$$

$$= \tan y + \cot x$$