

Math 111: Summary of Trigonometric Identities

Reciprocal Identities

$$\begin{aligned}\sin \theta &= \frac{1}{\csc \theta} & \cos \theta &= \frac{1}{\sec \theta} & \tan \theta &= \frac{1}{\cot \theta} \\ \csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta}\end{aligned}$$

Quotient Identities

$$\begin{aligned}\tan \theta &= \frac{\sin \theta}{\cos \theta} \\ \cot \theta &= \frac{\cos \theta}{\sin \theta}\end{aligned}$$

Pythagorean Identities

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \tan^2 \theta + 1 &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \csc^2 \theta\end{aligned}$$

Even/Odd Identities

$$\begin{aligned}\sin(-\theta) &= -\sin \theta & \csc(-\theta) &= -\csc \theta \\ \cos(-\theta) &= \cos \theta & \sec(-\theta) &= \sec \theta \\ \tan(-\theta) &= -\tan \theta & \cot(-\theta) &= -\cot \theta\end{aligned}$$

Cofunction Identities

$$\begin{aligned}\sin\left(\frac{\pi}{2} - \theta\right) &= \cos \theta & \cos\left(\frac{\pi}{2} - \theta\right) &= \sin \theta & \tan\left(\frac{\pi}{2} - \theta\right) &= \cot \theta \\ \csc\left(\frac{\pi}{2} - \theta\right) &= \sec \theta & \sec\left(\frac{\pi}{2} - \theta\right) &= \csc \theta & \cot\left(\frac{\pi}{2} - \theta\right) &= \tan \theta\end{aligned}$$

Sum and Difference Formulas

$$\begin{aligned}\sin(\alpha + \beta) &= \sin \alpha \cos \beta + \sin \beta \cos \alpha & \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \\ \sin(\alpha - \beta) &= \sin \alpha \cos \beta - \sin \beta \cos \alpha \\ \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta & \tan(\alpha - \beta) &= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \\ \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta\end{aligned}$$

Double Angle Identities

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \\ &= 1 - 2 \sin^2 \theta\end{aligned}$$

Power Reducing Identities

$$\begin{aligned}\sin^2 \theta &= \frac{1 - \cos 2\theta}{2} \\ \cos^2 \theta &= \frac{1 + \cos 2\theta}{2} \\ \tan^2 \theta &= \frac{1 - \cos 2\theta}{1 + \cos 2\theta}\end{aligned}$$

Half Angle Identities

$$\begin{aligned}\sin \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{2}} \\ \cos \frac{\theta}{2} &= \pm \sqrt{\frac{1 + \cos \theta}{2}} \\ \tan \frac{\theta}{2} &= \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}\end{aligned}$$

Sum to Product Identities

Product to Sum Identities

$$\begin{aligned}\sin \alpha \sin \beta &= \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)] \\ \cos \alpha \cos \beta &= \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)] \\ \sin \alpha \cos \beta &= \frac{1}{2} [\sin(\alpha - \beta) + \sin(\alpha + \beta)]\end{aligned}$$

$$\begin{aligned}\sin \alpha + \sin \beta &= 2 \sin \left(\frac{\alpha + \beta}{2}\right) \cos \left(\frac{\alpha - \beta}{2}\right) \\ \sin \alpha - \sin \beta &= 2 \cos \left(\frac{\alpha + \beta}{2}\right) \sin \left(\frac{\alpha - \beta}{2}\right) \\ \cos \alpha + \cos \beta &= 2 \cos \left(\frac{\alpha + \beta}{2}\right) \cos \left(\frac{\alpha - \beta}{2}\right) \\ \cos \alpha - \cos \beta &= -2 \sin \left(\frac{\alpha + \beta}{2}\right) \sin \left(\frac{\alpha - \beta}{2}\right)\end{aligned}$$