Example 3: You have 16 coins—a combination of pennies, nickels, and dimes worth $0.86. You have an equal number of pennies and dimes.

a. Write a system of equations *(no matrices)* that would help you determine how many pennies, nickels, and dimes you have.

$P + N + D = 16$

$.01P + .05N + .10D = .86$

$P = D$

$P + 0N - D = 0$

$P = 6$

$N = 4$

$D = 6$

$\Rightarrow P - D = 0$

b. Convert your system from part **a** into a matrix equation.

c. Use an inverse matrix and a graphing calculator to solve the equation.

Homework

1. At a carnival, different rides take different types of tokens. For $20, you could get any of the following packages:

14 gold, 20 silver, 24 bronze; 20 gold, 15 silver, 19 bronze; 30 gold, 5 silver, 13 bronze

a. Write a system of equations *(no matrices)* that would help you determine the value of each type of token.

$14G + 20S + 24B = 20$

$20G + 15S + 19B = 20$

$30G + 5S + 13B = 20$

b. Convert your system from part **a** into a matrix equation.

$$\begin{bmatrix} 14 & 20 & 24 \\ 20 & 15 & 19 \\ 30 & 5 & 13 \end{bmatrix} \begin{bmatrix} G \\ S \\ B \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix}$$

c. Use an inverse matrix and a graphing calculator to solve the equation.

Gold tokens cost .5 each; silver, .35 each; bronze, .25 each.

2. In the election of 1912, Woodrow Wilson, Teddy Roosevelt, and William Howard Taft were the only candidates to receive electoral votes—a total of 531. Wilson received the most electoral votes: he got 347 more than Roosevelt, who received 11 times as many votes as Taft.

a. Write a system of equations *(no matrices)* that would help you determine how many electoral votes each candidate received.

$$W + R + T = 531$$
$$W - R + 0T = 347$$
$$0W + R - 11T = 0$$

b. Convert your system from part **a** into a matrix equation.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -11 \end{bmatrix} \begin{bmatrix} W \\ R \\ T \end{bmatrix} = \begin{bmatrix} 531 \\ 347 \\ 0 \end{bmatrix}$$

c. Use an inverse matrix and a graphing calculator to solve the equation.

435 votes were cast for Wilson, 88 for Roosevelt, and 8 for Taft.

3. In a full day of sales, Regal Cinemas sold 2068 tickets and made $19,104. Tickets can be bought in three ways: a matinee ticket for $8.50, a student ticket (all day) for $8.00, or a regular ticket for $10.50. Based on past data, you believe that the number of regular-price tickets is probably 40% more than the number of matinee tickets.

a. Write a system of equations *(no matrices)* that would help you determine how many of each type of ticket were sold.

$$M + S + R = 2068$$
$$8.50M + 8S + 10.5R = 19{,}104$$
$$-1.4M + 0S + R = 0$$

b. Convert your system from part **a** into a matrix equation.

$$\begin{bmatrix} 1 & 1 & 1 \\ 8.5 & 8 & 10.5 \\ -1.4 & 0 & 1 \end{bmatrix} \begin{bmatrix} M \\ S \\ R \end{bmatrix} = \begin{bmatrix} 2068 \\ 19{,}104 \\ 0 \end{bmatrix}$$

c. Use an inverse matrix and a graphing calculator to solve the equation.

Regal Cinemas sold 640 matinee tickets, 532 student tickets, and 896 regular tickets.

4. In a room, there are some four-legged tables, some ears of corn, some people, and some dogs. (That must have been a weird party.) Altogether there are 508 legs, 198 ears, 180 eyes, and 150 items (tables + corn + people + dogs).

a. Write a system of equations *(no matrices)* that would help you determine how many of each item (tables, corn, people, dogs) there are.

$$4T + 2P + 4D + 0C = 508 \text{ legs}$$
$$0T + 2P + 2D + 1C = 198 \text{ ears}$$
$$0T + 2P + 2D + 0C = 180 \text{ eyes}$$
$$T + P + D + C = 150 \text{ items}$$

b. Convert your system from part **a** into a matrix equation.

$$\begin{bmatrix} 4 & 2 & 4 & 0 \\ 0 & 2 & 2 & 1 \\ 0 & 2 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} T \\ P \\ D \\ C \end{bmatrix} = \begin{bmatrix} 508 \\ 198 \\ 180 \\ 150 \end{bmatrix}$$

c. Use an inverse matrix and a graphing calculator to solve the equation.

$T = 42 \quad P = 10 \quad D = 80 \quad C = 18$