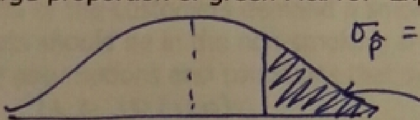
AP Statistics
Sample Proportions Practice

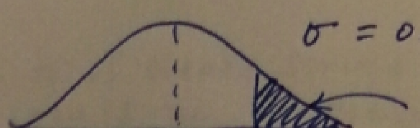
Name Key

Period ______ Date ______________

1. **M&M's.** The candy company claims that 10% of the M&M's it produces are green. Suppose that the candies are packaged at random in small bags containing about 50 M&M's. A class of elementary school students learning about percents opens several bags, counts the various colors of candies, and calculates the proportion that are green.
   a) If we plot a histogram showing the proportions of green candies in the various bags, what shape would you expect it to have?
      Symmetric
      $p = 0.1$
      $n = 50$
   b) Can that histogram be approximated by a Normal model? Explain. No
      $np = 50(.1) = 5$ $\quad n(1-p) = 50(.9) = 45$
   c) Where should the center of the histogram be?
      $\mu_{\hat{p}} = 0.1$
   d) What should the standard deviation of the proportion be?
      $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.1)(.9)}{50}} = 0.042$

2. Suppose the class buys bigger bags of candy, with 200 M&M's each. Again the students calculate the proportion of green candies they find.
   a) Explain why it's appropriate to use the Normal model to describe the distribution of the proportion of green M&M's they might expect.
      $np = 200(.1) = 20$ Both products are at least 10.
      $n(1-p) = 200(.9) = 180$
   b) Use the 68-95-99.7 Rule to describe how this proportion might vary from bag to bag. 68% of the bags should have a prop. of green M&M's w/in 1 st. dev. of the mean btw .079 and .121. 95% w/in 2 st. dev. 99.7 w/in 3 st. dev.
      $\sigma_{\hat{p}} = \sqrt{\frac{(.1)(.9)}{200}} = 0.021$ $\quad \mu_{\hat{p}} = 0.1$
   c) How would this model change if the bags contained even more candies?
      The standard deviation will decrease.
      As sample size ↑, variability ↓.

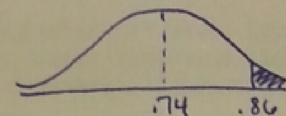
3. In a really large bag of M&M's, the students found 500 candies and 12% of them were green. Is this an unusually large proportion of green M&M's? Explain.
   $\sigma_{\hat{p}} = \sqrt{\frac{(.1)(.9)}{500}} = 0.013$ $\quad z = \frac{.12 - .1}{.013} = 1.53$
   
   .1 .12
   .062
   12% is 1.53 st. dev. from the mean. It's not an outlier but there is a small chance of finding a bag w/ a larger % of green M&M's.

4. **Loans.** Based on past experience, a bank believes that 7% of the people who receive loans will not make payments on time. The bank has recently approved 200 loans.
   a) What are the mean and standard deviation of the proportion of clients in this group who may not make timely payments?
      $\mu_{\hat{p}} = 0.07$ $\quad \sigma_{\hat{p}} = \sqrt{\frac{(.07)(.93)}{200}} = 0.018$
   b) What assumptions underlie your model? Are the conditions met? Explain.
      $N \geq 10(200)$ we are assuming the pop. is at least 2000 people
      $N \geq 2000$
      $np = 200(.07) = 14 \geq 10$
      $n(1-p) = 200(.93) = 186 \geq 10$
   c) What's the probability that over 10% of these clients will not make timely payments?
      $\sigma = 0.018$ $\quad z = \frac{.1 - .07}{.018} = 1.67$
      
      .07 .10
      $0.048 = \boxed{4.8\%}$

5. **Back to school.** Best known for its testing program. ACT, Inc., also complies data on a variety of issues in education. In 2004, the company reported that the national college freshman-to-sophomore retention rate held steady at 74% over the previous four years. Does a college where 522 of their 603 freshman returned the next year as sophomores have a right to brag that it has an unusually high retention rate? Explain.

$p = .74$ $\quad np = 603(.74) = 446 \geq 10$ $\quad \sigma_{\hat{p}} = \sqrt{\frac{(.74)(.26)}{603}} = 0.018$

$\hat{p} = \frac{522}{603} = 0.866$ $\quad n(1-p) = 603(.26) = 157 \geq 10$



$z = \frac{.86 - .74}{.018} = 6.67$

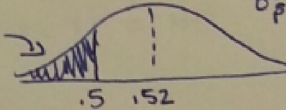
This school is over 6 st. dev. from the mean. We can say they have an usually high retention rate.

6. **Polling.** Just before a referendum on a school budget, a local newspaper polls 400 voters in an attempt to predict whether the budget will pass. Suppose that the budget actually has the support of 52% of the voters. What's the probability the newspaper's sample will lead them to predict defeat? Be sure to verify that the assumptions and conditions necessary for your analysis are met.

$n = 400$ $\quad N \geq 400(10)$ Assume Pop ≥ 4000 $\quad \sigma_{\hat{p}} = \sqrt{\frac{(.52)(.48)}{400}} = .025$

$p = .52$ $\quad np = 400(.52) = 208 \geq 10$

$n(1-p) = 400(.48) = 192 \geq 10$



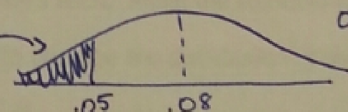
$z = \frac{.5 - .52}{.025} = -.8$

21.2% chance of getting a result below 50% which will make them think the budget won't pass.

7. **Apples.** When a truckload of apples arrives at a packing plant, a random sample of 150 is selected and examined for bruises, discoloration, and other defects. The whole truckload will be rejected if more than 5% of the sample is unsatisfactory. Suppose that in fact 8% of the apples on the truck do not meet the desired standard. What's the probability that the shipment is accepted anyway?

$n = 150$ $\quad np = 150(.08) = 12 \geq 10$ $\quad \sigma_{\hat{p}} = \sqrt{\frac{(.08)(.92)}{150}} = 0.022$

$p = .08$ $\quad n(1-p) = 150(.92) = 138 \geq 10$



$z = \frac{.05 - .08}{.022} = -1.36$

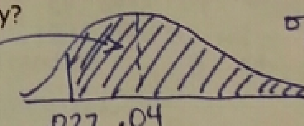
8.6% chance the shipment will be accepted.

8. **Genetic defect.** It's believed that 4% of children have a gene that may be linked to juvenile diabetes. Researches hoping to track 20 of these children for several years test 732 newborns for the presence of this gene. What's the probability that they find enough subjects for their study?

$n = 732$ $\quad np = 732(.04) = 29 \geq 10$ $\quad \sigma_{\hat{p}} = \sqrt{\frac{(.04)(.96)}{732}} = .007$

$p = .04$ $\quad n(1-p) = 732(.96) = 703 \geq 10$

$\hat{p} = \frac{20}{732} = .027$



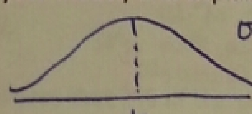
$z = \frac{.027 - .04}{.007} = -1.86$

96.8% chance of finding 20 children in a sample of 732 that have the gene.

9. **Nonsmokers.** While some nonsmokers do not mind being seated in a smoking section of a restaurant, about 60% of the customers demand a smoke-free area. A new restaurant with 120 seats is being planned. How many seats should be in the non-smoking area in order to be very sure of having enough seating there? Comment on the assumptions and conditions that support your model, and explain what "very sure" means to you.

$n = 120$ $\quad N \geq 10(120)$

$p = 0.6$ $\quad$ Assume pop ≥ 1200



$\sigma_{\hat{p}} = \sqrt{\frac{(.6)(.4)}{120}} = 0.044$

$np = 120(.6) = 72 \geq 10$

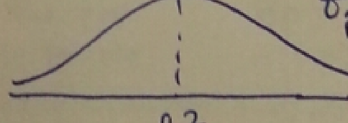
$n(1-p) = 120(.4) = 48 \geq 10$

Avg # of seats needed would be $0.6(120) = 72$. one st. dev. from that would be 77 seats. 2 st. dev. would be 82 seats.

10. **Meals.** A restaurateur anticipates serving about 180 people on a Friday evening, and believes that about 20% of the patrons will order the chef's steak special. How many of those meals should he plan on serving in order to be pretty sure of having enough steaks on hand to meet customer demand? Justify your answer, including an explanation of what "pretty sure" means to you.

$n = 180$ $\quad np = 180(.2) = 36 \geq 10$ $\quad \sigma_{\hat{p}} = \sqrt{\frac{(.2)(.8)}{180}} = 0.03$

$p = 0.2$ $\quad n(1-p) = 180(.8) = 144 \geq 10$



Avg. # of steaks served would be $(0.2)(180) = 36$. one st. dev. above that would be 41 steaks. 2 st. dev. would be 46 steaks.