These Rules are NOT Made to be Broken

Consider the following population, called X:

52 54 57 60 62 65 65 70 74 82

**1.** Find the mean and population standard deviation (Say you use L1 to do this). Record it in the table below.

**2.** Find the mean & standard deviation for each new data set in a–c.

(a) Add 8 to every number in the data set. (An easy way to do this is to set L2 equal to L1+8)

(b) Add 23 to every number in the data set (start with the original data set).

(c) Subtract 7 from every number in the data set (start with the original data set).

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| --- | --- | --- | --- |
| X | X + 8 | X + 23 | X - 7 |
| $$μ\_{x}= 64.1 $$ | $$μ\_{x+8}= 72.1 $$ | $$μ\_{x+23}= 87.1 $$ | $$μ\_{x-7}= 57.1 $$ |
| $$σ\_{x}= 8.8 $$ | $$σ\_{x+8}= 8.8 $$ | $$σ\_{x+23}= 8.8 $$ | $$σ\_{x-7}= 8.8 $$ |

**3.** Compare to the mean and standard deviation of the original data set. Write a rule for what happens to the mean and standard deviation of a data set if you add/subtract a certain number to each number in the data set.

 The mean changes by the same amount added or subtracted. The standard deviation does not change.

 $μ\_{x+a}=μ\_{x}+a$ $σ\_{x+a}=σ\_{x}$

**4.** Find the mean & standard deviation for each new data set in a–c.

(a) Multiply 6 by every number in the data set (start with the original data set).

(b) Multiply 15 by every number in the data set (start with the original data set).

(c) Multiply ½ by every number in the data set (start with the original data set).

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| X (from above) | 6X | 15X | $^{1}/\_{2}$X |
| $$μ\_{x}= 64.1 $$ | $$μ\_{6X}= 384.6 $$ | $$μ\_{15X}= 961.5 $$ | $$μ\_{^{1}/\_{2}x}= 32.05 $$ |
| $$σ\_{x}= 8.8 $$ | $$σ\_{6X}= 52.8 $$ | $$σ\_{15X}= 132 $$ | $$σ\_{^{1}/\_{2}x}= 4.4 $$ |

**5.** Compare to the mean and standard deviation of the original data set. Write a rule for what happens to the mean and standard deviation of a data set if you multiply a certain number by each number in the data set.

 The mean and standard deviation both change by the amount multiplied or divided.

 $μ\_{bx}=bμ\_{x}$ $σ\_{bx}=bσ\_{x}$ $σ\_{bx}^{2}=b^{2}σ\_{x}^{2}$

**6.** Find the mean & standard deviation for each new data set in a & b.

(a) Multiply every number in the data set by 5, then add 10 to each number (start with the original data set).

(b) Add 10 to each number in the data set, then multiply each number by 5 (start with the original data set).

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| X (from above) | 5X + 10 | 5(X + 10) |
| $$μ\_{x}= 64.1 $$ | $$μ\_{5x+10}= 330.5 $$ | $$μ\_{5(x+10)}= 370.5 $$ |
| $$σ\_{x}= 8.8 $$ | $$σ\_{5x+10}= 44 $$ | $$σ\_{5(x+10)}= 44.014 $$ |

**7.** Is the mean of 5X + 10 the same as the mean for 5(X + 10)? Is the standard deviation? No; Yes

**8.** Write a rule for what happens to the mean and standard deviation of a data set if you 1) multiply then add and

2) add then multiply.

 The mean changes in the same way each individual value changes. The st. dev. changes by the amount multiplied. Adding or subtracting an amount to each value does not change the st. dev.

 $μ\_{a+bx}=a+bμ\_{x}$ $σ\_{a+bx}=bσ\_{x}$

**9.** Consider the following populations:

X: 1 4 7

Y: 15 16 17

(a) Find the mean, standard deviation, and variance of data set X and data set Y. Record in the table below.

(b) Create a new variable, A, which is created by adding each number in data set X to each number in data set Y (Examples: 1 + 15, 1 + 16, 1 + 17, 4 + 15, etc). List every sum, even if it repeats (you can record them in a list in your calculator). The number of total outcomes should equal the number of outcomes in X times the number of total outcomes in Y. Find the mean, standard deviation, & variance of data set A.

(c) Create a new variable, B, which is created by subtracting each number in data set Y from each number in data set X (Examples: 1 – 15, 1 – 16, 1 – 17, 4 – 15, etc). List every difference, even if it repeats (you can record them in a list in your calculator). The number of total outcomes should equal the number of outcomes in X times the number of total outcomes in Y. Find the mean, standard deviation, and variance of data set B.

|  |  |  |  |
| --- | --- | --- | --- |
| X  | Y | A = X + Y | B = X - Y |
| $$μ\_{x}= 4 $$ | $$μ\_{y}= 16 $$ | $$μ\_{X+Y}= 20 $$ | $$μ\_{X-Y}= -12 $$ |
| $$σ\_{x}= 2.45 $$ | $$σ\_{y}= 0.816 $$ | $$σ\_{X+Y}= 2.58 $$ | $$σ\_{X-Y}= 2.58 $$ |
| $$σ\_{x}^{2}= 6 $$ | $$σ\_{y}^{2}= 0.67 $$ | $$σ\_{X+Y}^{2}= 6.67 $$ | $$σ\_{X-Y}^{2}= 6.67 $$ |

(d) Compare the means of X, Y, X + Y, and X – Y. Write a rule for what the mean will be for a random variable that is created by adding/subtracting two random variables together.

 The mean of the combined variables is the sum or difference of the individual means.

 $μ\_{x+y}=μ\_{x}+μ\_{y}$ $μ\_{x-y}=μ\_{x}-μ\_{y}$

(e) Compare the variance of X, Y, X + Y, and X – Y. Write a rule for what the variance will be for a random variable that is created by adding/subtracting two random variables together.

 The variance of the combined variables is the sum of the individual variances. Even if you want the difference between the variables, you still add the individual variances. YOU CAN ADD VARIANCES BUT NOT STANDARD DEVIATIONS.

 $σ\_{x+y}^{2}=σ\_{x}^{2}+σ\_{y}^{2}$ $σ\_{x-y}^{2}=σ\_{x}^{2}+σ\_{y}^{2}$

(f) Do the standard deviations follow the variance rule that you came up with in part (e)? Why do you think this is?

 No; this is because $\sqrt{x^{2}+y^{2}}\ne x+y$

**10.** Consider again the following population (same as in #9):

X: 1 4 7

(a) Enter the mean, standard deviation, and variance of data set X in the table below (already calculated in #9).

(b) Create a new variable, C, which is created by multiplying each number in X by 2. Find the mean, standard deviation, and variance of data set C. Enter them in the table.

(c) Create a new variable, D, which is created by adding each number in data set X to each number in data set X (Examples: 1 + 1, 1 + 4,…). List every sum, even if it repeats (you can record them in a list in your calculator). The number of total outcomes should equal (the number of outcomes in X)2. What do you expect the mean, variance, and standard deviation of data set D to be?

You might expect the mean, variance and standard deviations in the third column to be the same as those in the second column.

(d) Find the mean, standard deviation, and variance of data set D. Enter them in the table.

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| X  | C = 2X | D = X + X |
| $$μ\_{x}= 4 $$ | $$μ\_{2x}= 8 $$ | $$μ\_{X+X}= 8 $$ |
| $$σ\_{x}= 2.45 $$ | $$σ\_{2x}= 4.9 $$ | $$σ\_{X+X}= 3.46 $$ |
| $$σ\_{x}^{2}= 6 $$ | $$σ\_{2x}^{2}= 24 $$ | $$σ\_{X+X}^{2}= 11.97 $$ |

(e) How do the means of X, 2X, and X + X compare? Is this the mean of X + X you expected in part (c)? Yes

 $μ\_{2x}=2μ\_{x}=2\left(4\right)=8$ $μ\_{x+x}=μ\_{x}+μ\_{x}=4+4=8$

(f) How do the variances of X, 2X, and X + X compare? Is this the variance of X + X you expected in part (c)? No

 $σ\_{2x}^{2}=2^{2}σ\_{x}^{2}=4\left(6\right)=24$ $σ\_{x+x}^{2}=σ\_{x}^{2}+σ\_{x}^{2}=6+6=12$

(g) How do the standard deviations of X, 2X, and X + X compare? Is this the standard deviation of X + X you expected in part (c)? No

 $σ\_{2x}=2σ\_{x}=2\left(4.5\right)=4.9$ $σ\_{x+x}=\sqrt{σ\_{x}^{2}+σ\_{x}^{2}}=\sqrt{6+6}=3.46$

(h) Write a rule for what the mean will be for a random variable that is created by adding the random variable to itself.

 Adding a random variable to itself will double the mean. $μ\_{x+x}=μ\_{x}+μ\_{x}$

(i) Write a rule for what the variance and standard deviation will be for a random variable that is created by adding the random variable to itself.

 When you add a random variable to itself you may add the variances but not the standard deviations. To find the combined standard deviation, first add the variances and then take the square root.

 $σ\_{x+x}^{2}=σ\_{x}^{2}+σ\_{x}^{2}$ $σ\_{x+x}=\sqrt{σ\_{x}^{2}+σ\_{x}^{2}}$

(j) In the random variable world, is X + X = 2X? Explain.

 No. 2X only has 3 possible outcomes, but X + X has 9 possible outcomes.

Now let’s see if we can use these rules we’ve developed. Consider the following scenario:

**11.** Two different roads feed into a particular freeway entrance. Suppose that during a fixed time period, the number of cars coming from each road onto the freeway is a random variable with mean values as follows:

Road 1 2

Mean 800 1000

(a) What is the mean of the total number of cars entering the freeway? $μ\_{1+2}=μ\_{1}+μ\_{2}=800+1000=1800$

(b) Roadwork during a certain time period causes the traffic flow from road 2 to drop to 60% of its original value and traffic flow from road 1 doubles. What is the new mean for the total number of cars entering the freeway during the construction period?

 $μ\_{new}=μ\_{2x}+μ\_{0.6x}=2\left(800\right)+0.6\left(1000\right)=2200$

**12.** A certain college uses SAT scores as one of its admission criteria. The distribution of SAT scores among its entire population of applicants is:

SAT math score X: mean = 625, std. dev = 90

SAT verbal score Y: mean = 590, std. dev = 100

What are the mean and standard deviation of the total score X + Y among students applying to this college?

 $μ\_{total score}=μ\_{math}+μ\_{verbal}=625+590=1215$

 $σ\_{total score}=\sqrt{σ\_{math}^{2}+σ\_{verbal}^{2}}=\sqrt{90^{2}+100^{2}}=134.54$

**13.** A nationwide standardized exam consists of a multiple-choice section and a free response section. For each section, the mean and standard deviation are reported:

Multiple Choice: mean = 38, std. dev = 6

Free Response: mean = 30, std. dev = 7

If an examinee’s total score is computed by combining the multiple choice score with twice the free response score, what are the mean and standard deviation of the total score?

 $μ\_{total score}=μ\_{MC}+μ\_{2FR}=38+2\left(30\right)=98$

 $σ\_{total score}=\sqrt{σ\_{MC}^{2}+σ\_{2FR}^{2}}=\sqrt{6^{2}+\left(2^{2}\right)\left(7^{2}\right)}=15.232$