Roots & Polynomials Forwards & Backwards

Name __Key__

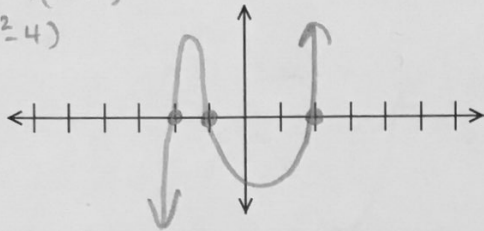Period _____   Date _____

Show work NEATLY on a separate sheet of paper!

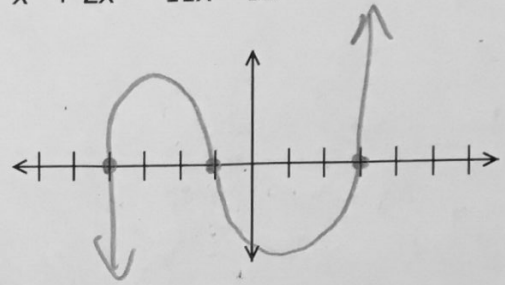Use any method to find *all* roots of the polynomial, then graph the polynomial on the given axes.

NOTE: Several of these are factorable!!!
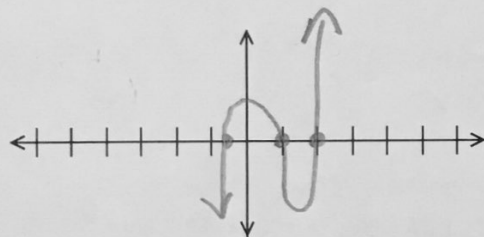
1. $x^3 + x^2 - 4x - 4$

$x^2(x+1) - 4(x+1)$

$(x+1)(x^2-4)$

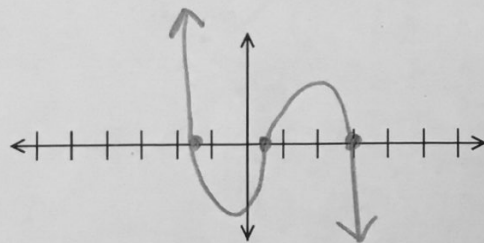**2.** $x^3 + 2x^2 - 11x - 12$

3. $2x^3 - 5x^2 + x + 2$

4. $-x^3 + 3x^2 + x - 3$

$-x^2+1 = 0$

$x^2 = 1$

$-x^2(x-3) + 1(x-3)$

$(-x^2+1)(x-3)$

5. $-4x^3 + 8x^2 + 15x - 9$
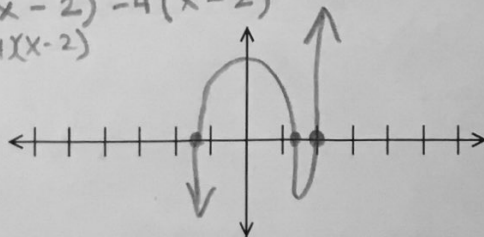(Hint: There is a whole-number positive root)

6. $x^4 + 6x^3 + 2x^2 - 12x$

$$-2 \begin{array}{|rrrrr} 1 & 6 & 2 & -12 & 0 \\ & -2 & -8 & 12 & 0 \\ \hline 1 & 4 & -6 & 0 & 0 \end{array}$$

$$0 \begin{array}{|rrrr} 1 & 4 & -6 & 0 \\ & 0 & 0 & 0 \\ \hline 1 & 4 & -6 & 0 \end{array}$$

$x^2 + 4x - 6$

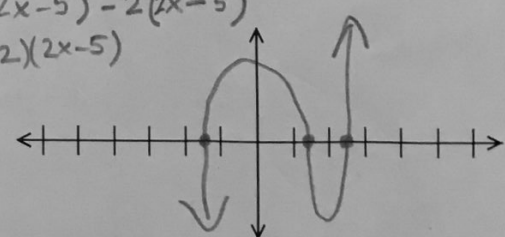$$\frac{-4 \pm \sqrt{4^2 - 4(1)(-6)}}{2} = \frac{-4 \pm \sqrt{40}}{2} = \frac{-4 \pm 2\sqrt{10}}{2}$$

7. $2x^3 - 4x^2 - 4x + 8$

$2x^2(x-2) - 4(x-2)$

$(2x^2-4)(x-2)$

$2x^2 = 0$

$2x = 4$

$x^2 = 2$

$x = \pm\sqrt{2}$

8. $2x^3 - 5x^2 - 4x + 10$

$x^2(2x-5) - 2(2x-5)$

$(x^2-2)(2x-5)$

Show work NEATLY on a separate sheet of paper!

Write a polynomial *in standard* form that fits the given description.

9. The only roots are
   $x = 3, -2,$ and 1

   $(x-3)(x+2)(x-1)$

10. The only roots are
   $x = 5$ (multiplicity 1) and -4 (multiplicity 2)

   $(x-5)(x+4)^2$

11. The only roots are
   $x = 5, -3, \frac{1}{2},$ and $\frac{3}{4}$
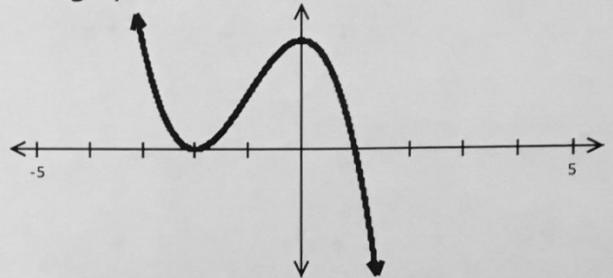   The *y*-intercept is 90

   $90 = a(x-5)(x+3)(2x-1)(4x-3)$
   $90 = a(-5)(3)(-1)(-3)$
   $90 = -45a$
   $-2 = a$
   $y = -2(x-5)(x+3)(2x-1)(4x-3)$

12. The graph is shown below:

   $-(x+2)^2(x-1)$

13. The polynomial has roots
   $x = -3, 4+\sqrt{2},$ and $\underline{4-\sqrt{2}}$
   (What other root *must* it have?)

   $(x+3)\ (x^2-8x+14)$

   $x = 4 \pm \sqrt{2}$
   $x-4 = \pm\sqrt{2}$
   $(x-4)^2 = 2$
   $(x-4)^2 - 2 = 0$
   $(x^2-8x+16)-2 = 0$
   $x^2-8x+14 = 0$

14. The graph is shown below.
   (Note that the *y*-axis has a scale this time!)
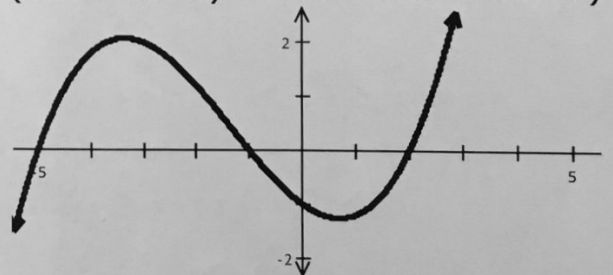
   $-1 = a(x+5)(x+1)(x-2)$
   $-1 = a(5)(1)(-2)$
   $-1 = -10a$
   $\frac{1}{10} = a$

   $y = \frac{1}{10}(x+5)(x+1)(x-2)$

15. The polynomial has roots
   $x = \sqrt{3}, 0, 4,$ and $\underline{-\sqrt{3}}$

   $x(x-4)(x^2-3)$

   $x = \pm\sqrt{3}$
   $x^2 = 3$
   $x^2-3 = 0$

16. The polynomial has roots
   $x = 1, 3+\sqrt{6},$ and $\underline{3-\sqrt{6}}$
   The *y*-intercept is 12.

   $12 = a(x-1)(x^2-6x+3)$
   $12 = a(-1)(3)$
   $12 = -3a$
   $-4 = a$

   $x = 3\pm\sqrt{6}$
   $x-3 = \sqrt{6}$
   $(x-3)^2 = 6$
   $(x-3)^2 - 6 = 0$
   $(x^2-6x+9)-6 = 0$
   $x^2-6x+3 = 0$

   $y = -4(x-1)(x^2-6x+3)$