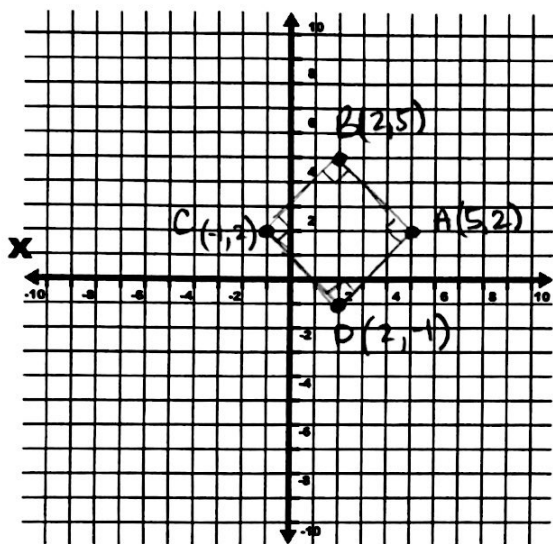


You are told that ABCD is a parallelogram. Use the properties of quadrilaterals, along with slope, midpoint and distance, to determine if ABCD is a rectangle, rhombus, square, or just a parallelogram. Justify your answer (show your work).

1. A(5, 2), B(2, 5), C(-1, 2), D(2, -1)

Square

$$\overline{AB} = \overline{BC} = \overline{CD} = \overline{DA}$$

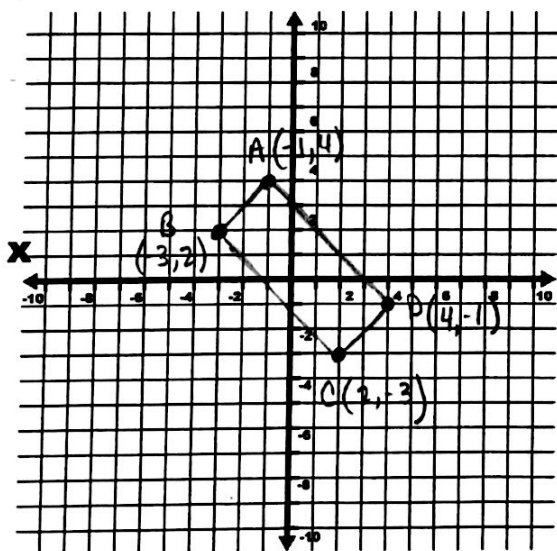


slope of AB = $\frac{2-5}{5-2} = -\frac{3}{3} = -1$
 slope of BC = $\frac{5-2}{2-(-1)} = \frac{3}{3} = 1$
 slope of CD = $\frac{-1-2}{2-(-1)} = -\frac{3}{3} = -1$
 slope of DA = $\frac{2-(-1)}{5-2} = \frac{3}{3} = 1$

3. A(-1, 4), B(-3, 2), C(2, -3), D(4, -1)

$$BC = AD = \sqrt{(2-(-3))^2 + (-3-2)^2} = \sqrt{25+25} = 5\sqrt{2}$$

Rect.

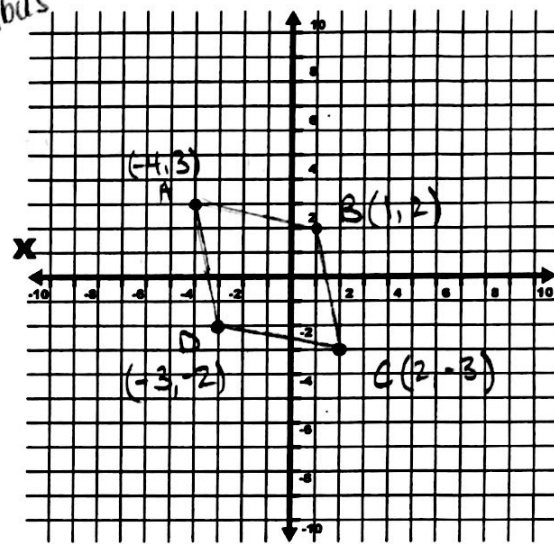


slope of AB = slope of CD = 1
 slope of BC = slope of AD = -1
 sides are \perp

$$AB = CD = \sqrt{(-1-(-3))^2 + (4-2)^2} = \sqrt{4+4} = 2\sqrt{2}$$

2. A(-4, 3), B(1, 2), C(2, -3), D(-3, -2)

rhombus

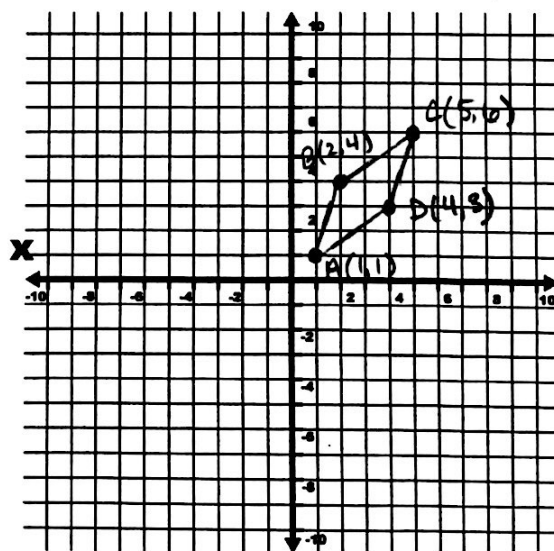


slope of AB = $\frac{2-3}{1-(-4)} = -\frac{1}{5}$
 slope of BC = $\frac{-3-2}{2-1} = -\frac{5}{1} = -5$
 slopes not \perp

$$AB = BC = CD = DA = \sqrt{(-4-1)^2 + (3-2)^2} = \sqrt{25+1} = \sqrt{26}$$

4. A(1, 1), B(2, 4), C(5, 6), D(4, 3)

parallelogram



slope of AB = slope of CD = $\frac{3}{1} = 3$
 slope of BC = slope of AD = $\frac{2}{3}$

AB \parallel CD
 BC \parallel AD

5. Plot points A(-3, -1), B(-1, 2), C(4, 2), D(2, -1).

- a) Draw the diagonals of ABCD. Find the coordinates of the midpoint of each diagonal. What do you notice?

$$\text{midpoint of AC} = \left(\frac{-3+4}{2}, \frac{-1+2}{2} \right) = \left(\frac{1}{2}, \frac{1}{2} \right)$$

$$\text{midpoint of BD} = \left(\frac{-1+2}{2}, \frac{2-1}{2} \right) = \left(\frac{1}{2}, \frac{1}{2} \right)$$

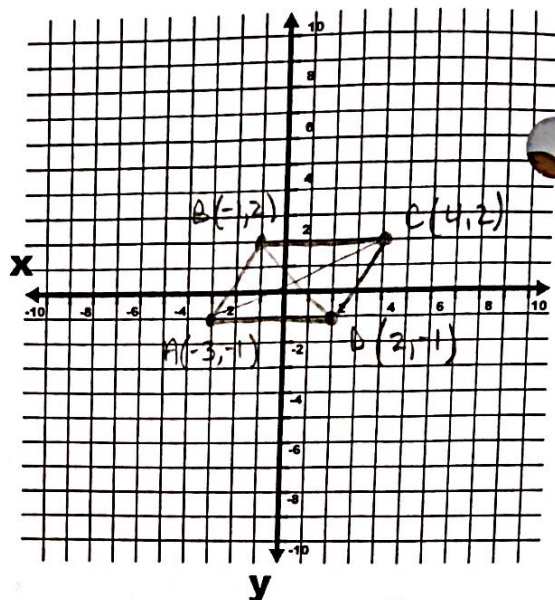
They are the same
They bisect each other

- b) Find the slopes of the diagonals. What do you notice?

$$\text{slope of AC} = \frac{3}{7}$$

not \perp

$$\text{slope of BD} = \frac{-3}{3} = -1$$



- c) Based on your answers to parts b and c, what type of quadrilateral is figure ABCD?

Parallelogram

6. Plot points P(4, 1), W(-2, 3), K(-6, -4), M(2, -5).

- a) Draw the diagonals of PWKM. Find the coordinates of the midpoint of each diagonal. What do you notice?

$$\text{midpoint of WM} = \left(\frac{-2+2}{2}, \frac{3-5}{2} \right) = (0, -1)$$

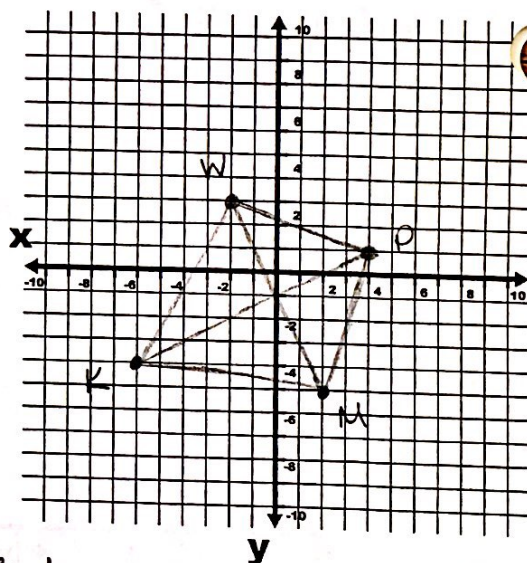
$$\text{midpoint of KP} = \left(\frac{4+(-6)}{2}, \frac{1-4}{2} \right) = (-1, -\frac{3}{2})$$

They are not the same.
They do not bisect each other.

- b) Find the slopes of the diagonals. What do you notice?

$$\text{slope of KP} = \frac{1-(-4)}{4-(-6)} = \frac{5}{10} = \frac{1}{2}$$

$$\text{slope of WM} = \frac{-5-3}{2-(-2)} = \frac{-8}{4} = -2 \quad \text{They are } \perp$$



- c) Find the lengths of the diagonals. What do you notice?

$$\overline{KP} = \sqrt{(4-(-6))^2 + (1-(-4))^2} = \sqrt{100+25} = \sqrt{125} = 5\sqrt{5}$$

$$\overline{WM} = \sqrt{(-2-2)^2 + (3-(-5))^2} = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$$

$$\begin{aligned} &125 \\ &\wedge \\ &5 \sqrt{25} \\ &\wedge \\ &5 \sqrt{5} \\ &80 \\ &\wedge \\ &2 \sqrt{40} \\ &\wedge \\ &2 \sqrt{20} \\ &\wedge \\ &2 \sqrt{10} \\ &\wedge \\ &2 \sqrt{5} \end{aligned}$$

- d) Based on your answers to parts b and c, what type of quadrilateral is figure PWKM?

Kite

7. Plot points A(1, 0), B(-1, 2), and C(2, 5).

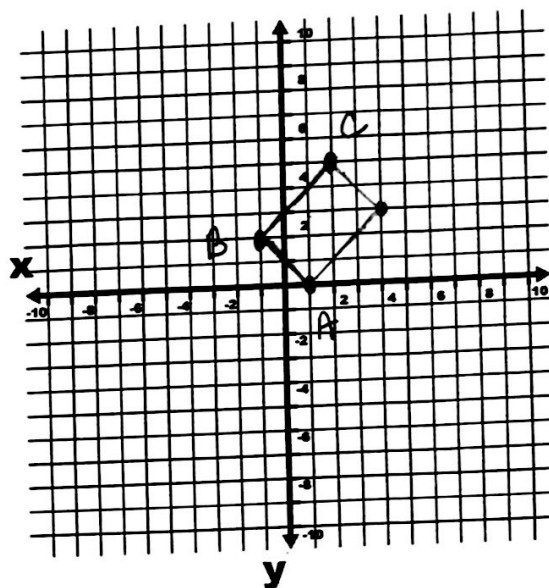
a) Find the coordinates of a 4th point, D, that would make ABCD a rectangle. Justify your answer (show your work).

$$\text{slope of AB} = -1$$

$$\text{slope of BC} = 1$$

$$D(4, 3)$$

In order for CD to have same slope and length as AB, point D must be @ (4, 3). This will also give BC and AD the same slope and length. This will also give 4 right angles.



b) Find the coordinates of a 4th point, D, that would make ABCD a parallelogram that is not also a rectangle. Justify your answer (show your work).

If point D were @ (0, 7), this will cause $AB \parallel CD$ and $BD \parallel AC$

