9-2 Enrichment
Folium

In Lesson 9-2 you learned how to graph a variety of equations written in polar form. In particular, you were introduced to the rose curves whose graphs can have three or more equal loops or petals. Let's now look at another leafed curve known as a *folium*. The Cartesian equation and polar equation of a folium is given below.

Cartesian equation: $(x^2 + y^2)[\, y^2 + x(x + b)] = 4axy^2$

Polar equation: $r = -b \cos \theta + 4a \cos \theta \sin^2 \theta$

There are three forms of the folium: the simple folium, the double folium, and the trifolium. When $b = 4a$ in the polar equation given above we have a simple folium; when $b = 0$, we have a double folium; and when $b = a$, we have a trifolium. Let's find the general equation of a simple folium.

$r = -b \cos \theta + 4a \cos \theta \sin^2 \theta$, where $b = 4a$

$\quad = -b \cos \theta + 4a \cos \theta \sin^2 \theta$

$\quad = -4a \cos \theta + 4a \cos \theta \sin^2 \theta$

$\quad = 4a \cos \theta (\sin^2 \theta - 1)$

To graph a simple folium, we must first choose a value for a. Suppose $a = 1$; our equation would be $r = 4 \cos \theta (\sin^2 \theta - 1)$. The graph of our simple folium is shown to the right.
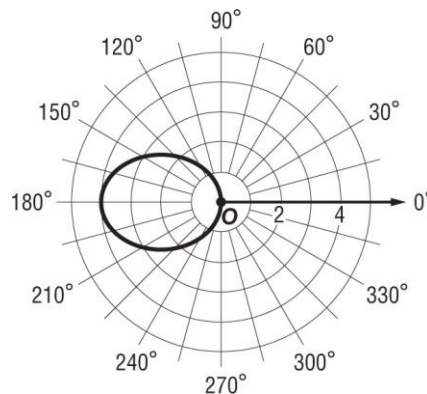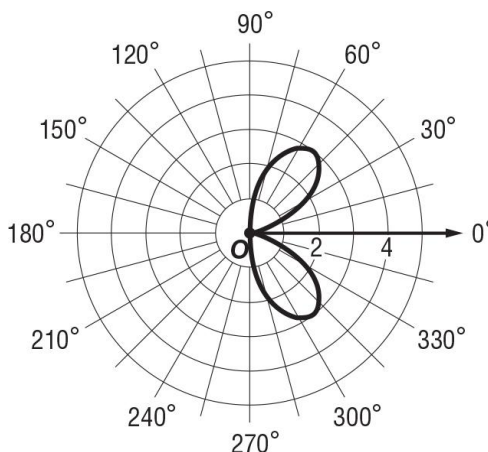
Exercises

1. Find the general equation of a double folium.

$r = 4a \cos \theta \sin^2 \theta$

2. Write the equation of a double folium where $a = 2$.

$r = 8 \cos \theta \sin^2 \theta$

3. Graph your equation from Exercise 2.

4. Find the general equation of a trifolium.

$r = a \cos \theta (4 \sin^2 \theta - 1)$

5. Write the equation of a trifolium where $a = 1$.

$r = \cos \theta (4 \sin^2 \theta - 1)$

6. Graph your equation from Exercise 5.