# 9-3 Enrichment
Points of Intersection

In this course and in other mathematics courses, you were exposed to the idea of solving systems of equations. Graphically this means to find the point(s) of intersection of the graphs of the equations found in the system. This same idea can be applied to a system of polar equations. Suppose you are given the following system and are asked to find all points that satisfy both equations over the interval $[0, 2\pi)$, or in other words, the points of intersection of their graphs.

$$r = \cos \theta$$
$$r = 1$$

First let's graph the system. The larger circle is the graph of $r = 1$ and the smaller circle is the graph of $r = \cos \theta$. As you can see, they appear to intersect in one point. Possibly $(1, 0)$? Let's prove this algebraically.
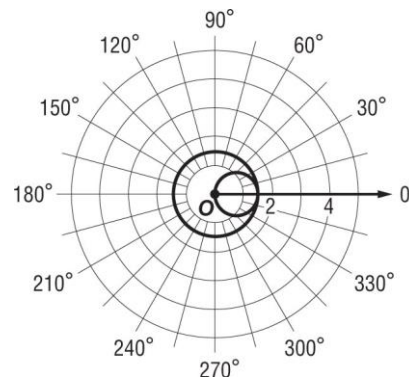
Since the left sides of both equations equal r, you can set the right sides equal to one another obtaining the equation $\cos \theta = 1$. Solving this equation yields $\theta = 0$. Thus, our point of intersection is indeed $(1, 0)$.

Systems often have more than one solution. Consider the following system of equations. Solve the system.

$$r = \sin \theta$$
$$r = \frac{1}{2}$$

Set the right side of each equation equal to one another.

$$\sin \theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

Thus, the points of intersection are $\left(\frac{1}{2}, \frac{\pi}{6}\right)$ and $\left(\frac{1}{2}, \frac{5\pi}{6}\right)$.

Exercises

Find all points of intersection of the given curves over the interval $[0, 2\pi)$.

1. $r = \sqrt{2}, r = 2 \sin \theta$

$$\left(\sqrt{2}, \frac{\pi}{4}\right), \left(\sqrt{2}, \frac{3\pi}{4}\right)$$

2. $r = 3, r = \cos \theta + 3$

$$\left(3, \frac{\pi}{2}\right), \left(3, \frac{3\pi}{2}\right)$$

3. $r = \sqrt{3}, r = 2 \cos \theta$

$$\left(\sqrt{3}, \frac{\pi}{6}\right), \left(\sqrt{3}, \frac{11\pi}{6}\right)$$

4. $r = 1, r = \sin \theta + 1$

$$(1, 0), (1, \pi)$$