

(23)

$$\begin{aligned} & \csc x - \cos x \cot x \\ &= \frac{1}{\sin x} - \left(\frac{\cos x}{1}\right) \left(\frac{\cos x}{\sin x}\right) \\ &= \frac{1}{\sin x} - \frac{\cos^2 x}{\sin x} \\ &= \frac{1 - \cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \sin x \end{aligned}$$

(25)

$$\begin{aligned} & \frac{\tan x + \sin x \sec x}{\csc x \tan x} = \frac{\frac{\sin x}{\cos x} + \left(\frac{\sin x}{1}\right) \left(\frac{1}{\cos x}\right)}{\left(\frac{1}{\sin x}\right) \left(\frac{\sin x}{\cos x}\right)} \\ &= \frac{\frac{\sin x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{2 \sin x}{\cos x} \cdot \cos x \\ &= \frac{2 \sin x \cos x}{\cos x} = 2 \sin x \end{aligned}$$

(27)

$$\begin{aligned} & \frac{\csc x \cos x + \cot x}{\sec x \cot x} = \frac{\left(\frac{1}{\sin x}\right) (\cos x) + \left(\frac{\cos x}{\sin x}\right)}{\left(\frac{1}{\cos x}\right) \left(\frac{\cos x}{\sin x}\right)} \\ &= \frac{\frac{\cos x}{\sin x} + \frac{\cos x}{\sin x}}{\frac{1}{\sin x}} = \frac{2 \cos x}{\sin x} \cdot \sin x \\ &= \frac{2 \cos x \sin x}{\sin x} = 2 \cos x \end{aligned}$$

(29)

$$\frac{\sec^2 x}{\cot^2 x + 1} = \frac{\sec^2 x}{\csc^2 x} = \left[\frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} \right]$$

$$= \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

(33)

$$\left(\frac{1+\cos x}{1+\cos x} \right) \frac{1-\cos x}{\tan x} + \frac{\sin x}{1+\cos x} \left(\frac{\tan x}{\tan x} \right)$$

FOIL this

$$= \frac{(1+\cos x)(1-\cos x) + \sin x \tan x}{(\tan x)(1+\cos x)}$$

$$= \frac{(1 - \cancel{\cos x} + \cancel{\cos x} - \cos^2 x) + \sin x \tan x}{(\tan x)(1+\cos x)}$$

$$= \frac{(1 - \cos^2 x) + \sin x \tan x}{\tan x (1+\cos x)}$$

$$= \frac{\sin^2 x + \sin x \tan x}{\tan x (1+\cos x)} = \frac{\sin^2 x + \left(\frac{\sin x}{1} \right) \left(\frac{\sin x}{\cos x} \right)}{\tan x (1+\cos x)}$$

$$= \frac{\left(\frac{\cos x}{\cos x} \right) \frac{\sin^2 x}{1} + \frac{\sin^2 x}{\cos x}}{\tan x (1+\cos x)} = \left[\frac{\frac{\cos x \sin^2 x + \sin^2 x}{\cos x}}{\tan x (1+\cos x)} \right]$$

$$= \frac{\cos x \sin^2 x + \sin^2 x}{\cos x \tan x (1+\cos x)} = \frac{\sin^2 x (\cancel{\cos x} + 1)}{\left(\frac{\cos x}{1} \right) \left(\frac{\sin x}{\cos x} \right) (1+\cos x)} = \frac{\sin^2 x}{\cancel{\sin x}} = \sin x$$