A Horse Race Expected Value

Name Key

Period _____ Date _____

Thanks to you, the betting office has realized some horses are more likely to win, so they will pay betters less if those horses win. (Way to go, party pooper!) Here are the new payouts:

Horse #	1	2	3	4	5	6	7	8	9	10	11	12
Payout	$200	$70	$35	$22	$16	$14	$10	$14	$16	$22	$35	$70

1. If you bet on horse 4, the *theoretical* probability you will win is $\frac{3}{36}$ (refer to **#10** on the other sheet).

   a. So there is a $\frac{3}{36}$ probability you'll win $22, and a $\frac{33}{36}$ probability you'll win $0. What are your expected earnings if you bet on horse 4?

   $\left(\frac{3}{36}\right)(22) + \left(\frac{33}{36}\right)(0) = \$1.83$

   b. But of course it costs money to place a bet! At this racetrack, it costs $2 to place a bet on any horse. *Now* what are your expected earnings/losses if you bet on horse 4?

   $\$1.83 - \$2 = \$-0.17$

2. Calculate the expected earnings/losses on a $2 bet for each horse. (You just did it for horse 4.) Refer to **#10** on the other sheet for P(win).

Horse #	1	2	3	4	5	6	7	8	9	10	11	12
P(win) **#10** on other sheet	0	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$
Winning Payout	$200	$70	$35	$22	$16	$14	$10	$14	$16	$22	$35	$70
P(lose)	1	$\frac{35}{36}$	$\frac{34}{36}$	$\frac{33}{36}$	$\frac{32}{36}$	$\frac{31}{36}$	$\frac{30}{36}$	$\frac{31}{36}$	$\frac{32}{36}$	$\frac{33}{36}$	$\frac{34}{36}$	$\frac{35}{36}$
Losing Payout	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0
Exp. Val. if you bet on this horse	$-2	$-.06	$-.06	$-.17	$-.22	$-.06	$-.33	$-.06	$-.22	$-.17	$-.06	$-.06

(Remember, it costs $2 to place a bet!)

3. If you were going to bet on this race a million times (always picking the same horse)…

   a. Which horse is the best (or least bad) choice? There may be more than one. 2, 3, 6, 8, 11, 12

   b. Which horse is the worst choice? There may be more than one. 1

   c. Should you go through with your plan to bet a million times? Explain *mathematically* in two or three full sentences.

   No. The expected value for each horse is negative which means you will lose money in the long run with each horse.

More Practice:

4. You have a choice of investing in two projects:
Project A will result in a loss of $26,000 with a probability of .30, breaking even with a probability of .50, or a gain of $68,000 with a probability of .20.
Project B will result in a loss of $71,000 with a probability of .20, breaking even with a probability of .65, or a gain of $143,000 with a probability of .15.

Which investment is better? Justify your answer.

A: $-26{,}000(.3) + 0(.5) + 68{,}000(.2) = \$5800$

B: $-71{,}000(.2) + 0(.65) + 143{,}000(.15) = \$7250$

B is better. B has a higher expected value.

5. A men's soccer team plays soccer 0, 1, or 2 days a week. The probability that they play 0 days is 0.2, the probability that they play 1 day is 0.5, and the probability that they play 2 days is 0.3. Find the long-term average, μ, or expected value of the days per week the men's soccer team plays soccer.

# of days	0	1	2
prob.	0.2	0.5	0.3

Exp value $= (0).2 + (1).5 + (2).3 = 1.1$ days

6. Suppose you play a game of chance in which you choose 5 numbers from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. You may choose a number more than once. You pay $2 to play and could profit $100,000 if you match all 5 numbers in order (you get your $2 back plus $100,000). Over the long term, what is your *expected* profit of playing the game?

10 10 10 10 10 = 100,000 choices

outcomes	Win $100,000	Lose
prob.	$\frac{1}{100{,}000}$	$\frac{99{,}999}{100{,}000}$

Exp profit $= \frac{1}{100{,}000}(100{,}000) + \frac{99{,}999}{100{,}000}(0) = \$1 - \$2 = -\$1$

7. Suppose you play a game with a biased coin. You play each game by tossing the coin once. P(heads) = 2/3 and P(tails)= 1/3. If you toss a head, you pay $6. If you toss a tail, you win $10. If you play this game many times, will you come out ahead?

Outcomes	Heads (pay $6)	Tails (win $10)
prob.	$\frac{2}{3}$	$\frac{1}{3}$

Exp value $= (\frac{2}{3})(-6) + (\frac{1}{3})(10)$

$= \$-0.67$ you will lose money.

8. The Green Mountain Lottery in Vermont allows you to play a three digit number (0 to 9) and repeats are allowed. If you win, the prize is $500. What is the expected value of your winnings?

10 10 10 = 1,000 choices

outcomes	Win $500	Lose
prob.	$\frac{1}{1000}$	$\frac{999}{1000}$

Exp. value $= \frac{1}{1000}(500) + \frac{999}{1000}(0)$

$= \$0.50$

9. Suppose a fair coin is tossed three times and we let x= number of heads. Find $\mu = E(x)$.

outcomes	0	1	2	3
prob.	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$E(X) = 0(\frac{1}{8}) + 1(\frac{3}{8}) + 2(\frac{3}{8}) + 3(\frac{1}{8})$

$= 1.5$