

Verifying Identities

Date _____ Period _____

Verify each identity.

$$1) \frac{\sin x + \csc x}{\csc x} = \sin^2 x + 1$$

$$2) \cot x - \sin x = \frac{\cos x - \sin^2 x}{\sin x}$$

$$3) \frac{1 - \csc x}{\csc x} = \sin x - 1$$

$$4) \frac{1}{1 + \tan x} = \frac{\cos x}{\cos x + \sin x}$$

$$5) \cot^2 x(1 + \cot^2 x) = \frac{1}{\tan^2 x \sin^2 x}$$

$$6) \frac{\sec x}{\cot^2 x - \csc^2 x} = \frac{\tan^2 x - \sec^2 x}{\cos x}$$

$$7) \frac{\csc^2 x + \sec^2 x}{\sec x} = \frac{1}{\sin^2 x \cos x}$$

$$8) \csc x \sin x + \cot^2 x = \frac{\csc x}{\sin x}$$

$$9) -\csc^2 x \cos^2 x = 1 - \csc^2 x$$

$$10) \sin x \csc x + \tan^2 x = \sec^2 x$$

Answers to Verifying Identities

1) $\frac{\sin x + \csc x}{\csc x}$ Decompose into sine and cosine

$$\frac{\sin x + \frac{1}{\sin x}}{\frac{1}{\sin x}} \quad \text{Simplify}$$

$$\sin^2 x + 1 \quad \blacksquare$$

3) $\frac{1 - \csc x}{\csc x}$ Decompose into sine and cosine

$$\frac{1 - \frac{1}{\sin x}}{\frac{1}{\sin x}} \quad \text{Simplify}$$

$$\sin x - 1 \quad \blacksquare$$

5) $\cot^2 x(1 + \cot^2 x)$ Use $\cot^2 x + 1 = \csc^2 x$

$$\cot^2 x \csc^2 x \quad \text{Use } \csc x = \frac{1}{\sin x}$$

$$\frac{\cot^2 x}{\sin^2 x} \quad \text{Use } \cot x = \frac{1}{\tan x}$$

$$\frac{1}{\tan^2 x \sin^2 x} \quad \blacksquare$$

2) $\cot x - \sin x$ Decompose into sine and cosine

$$\frac{\cos x}{\sin x} - \sin x \quad \text{Simplify}$$

$$\frac{\cos x - \sin^2 x}{\sin x} \quad \blacksquare$$

4) $\frac{1}{1 + \tan x}$ Decompose into sine and cosine

$$\frac{1}{1 + \frac{\sin x}{\cos x}} \quad \text{Simplify}$$

$$\frac{\cos x}{\cos x + \sin x} \quad \blacksquare$$

6) $\frac{\sec x}{\cot^2 x - \csc^2 x}$ Use $\cot^2 x + 1 = \csc^2 x$

$$\frac{\sec x}{-1} \quad \text{Decompose into sine and cosine}$$

$$\frac{1}{\cos x} \quad \text{Simplify}$$

$$-\frac{1}{\cos x} \quad \text{Use } \tan^2 x + 1 = \sec^2 x$$

$$\frac{\tan^2 x - \sec^2 x}{\cos x} \quad \blacksquare$$

7) $\frac{\csc^2 x + \sec^2 x}{\sec x}$ Decompose into sine and cosine

$$\frac{\left(\frac{1}{\sin x}\right)^2 + \left(\frac{1}{\cos x}\right)^2}{\frac{1}{\cos x}}$$

Simplify

$$\frac{\cos^2 x + \sin^2 x}{\sin^2 x \cos x}$$

Use $\sin^2 x + \cos^2 x = 1$

$$\frac{1}{\sin^2 x \cos x}$$

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8) $\csc x \sin x + \cot^2 x$ Decompose into sine and cosine

$$\frac{1}{\sin x} \cdot \sin x + \left(\frac{\cos x}{\sin x}\right)^2$$

Simplify

$$\frac{\sin^2 x + \cos^2 x}{\sin^2 x}$$

Use $\sin^2 x + \cos^2 x = 1$

$$\frac{1}{\sin^2 x}$$

Use $\csc x = \frac{1}{\sin x}$

$$\frac{\csc x}{\sin x}$$

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9) $-\csc^2 x \cos^2 x$ Use $\csc x = \frac{1}{\sin x}$

$$-\frac{\cos^2 x}{\sin^2 x}$$

Use $\cot x = \frac{\cos x}{\sin x}$

$$-\cot^2 x$$

Use $\cot^2 x + 1 = \csc^2 x$

$$1 - \csc^2 x$$

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$$10) \sin x \csc x + \tan^2 x$$

Decompose into sine and cosine

$$\sin x \cdot \frac{1}{\sin x} + \left(\frac{\sin x}{\cos x}\right)^2$$

Simplify

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

Use $\sin^2 x + \cos^2 x = 1$

$$\frac{1}{\cos^2 x}$$

Use $\sec x = \frac{1}{\cos x}$

$$\sec^2 x$$

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