

## Part 1: Multiple Choice. Circle the letter corresponding to the best answer.

1. You want to compute a 90% confidence interval for the mean of a population with unknown population standard deviation. The sample size is 30. The value of  $t^*$  you would use for this interval is
- (a) 1.96  
(b) 1.645  
(c) 1.699  
(d) 0.90  
(e) 1.311  
(f) None of the above
2. A polling organization announces that the proportion of American voters who favor congressional term limits is 64%, with a 95% confidence margin of error of 3%. This means:
- (a) If the poll were conducted again in the same way, there is a 95% chance that the fraction of voters favoring term limits in the second poll would be between 61% and 67%.  
(b) There is a 95% probability that the true percent of voters favoring term limits is between 61% and 67%.  
(c) If the poll were conducted again the same way, there is a 95% probability that the percent of voters favoring term limits in the second poll would be within 3% of the percent favoring term limits in the first poll.  
(d) Among 95% of the voters, between 61% and 67% favor term limits.  
(e) None of the above.
3. The college newspaper of a large Midwestern university periodically conducts a survey of students on campus to determine the attitude on campus concerning issues of interest. Pictures of the students interviewed along with quotes of their responses are printed in the paper. Students are interviewed by a reporter "roaming" the campus selecting students to interview "haphazardly." On a particular day the reporter interviews five students and asks them if they feel there is adequate student parking on campus. Four of the students say, "no." Which of the following conditions for inference about a proportion using a confidence interval are violated in this example?
- (a) The data are an SRS from the population of interest. ← violated  
(b) The population is at least ten times as large as the sample.  
(c)  $n\hat{p} \geq 10$  and  $n(1 - \hat{p}) \geq 10$ . ← violated  
(d) We are interested in inference about a proportion.  
(e) More than one condition is violated.
4. To assess the accuracy of a laboratory scale, a standard weight that is known to weigh 1 gram is repeatedly weighed a total of  $n$  times and the mean  $\bar{x}$  of the weighings is computed. Suppose the scale readings are Normally distributed with unknown mean  $\mu$  and standard deviation  $\sigma = 0.01$  g. How large should  $n$  be so that a 95% confidence interval for  $\mu$  has a margin of error of  $\pm 0.0001$ ?
- (a) 100                      (b) 196                      (c) 27,061                      (d) 10,000                      (e) 38,416

$$0.0001 = 1.96 \left( \frac{0.01}{\sqrt{n}} \right)$$

5. A 95% confidence interval for the mean reading achievement score for a population of third-grade students is (44.2, 54.2). Suppose you compute a 99% confidence interval using the same information. Which of the following statements is correct?

- (a) The intervals have the same width.  
 (b) The 99% interval is shorter.  
 (c) The 99% interval is longer.  
 (d) The answer can't be determined from the information given.  
 (e) None of the above. The answer is \_\_\_\_\_.

6. A random sample of 900 individuals has been selected from a large population. It was found that 180 are regular users of vitamins. Thus, the proportion of the regular users of vitamins in the population is estimated to be 0.20. The standard error of this estimate is approximately

- (a) 0.1600  
 (b) 0.0002  
 (c) 0.4000  
 (d) 0.0133  
 (e) 0.0267

$$SE = \sqrt{\frac{.2(.8)}{900}}$$

7. The effect of acid rain upon the yield of crops is of concern in many places. In order to determine baseline yields, a sample of 13 fields was selected, and the yield of barley (g/400 m<sup>2</sup>) was determined. The output from SAS appears below:

		QUANTILES(DEF=4)				EXTREMES			
N	13	SUM WGTs	13	100%MAX	392	99%	392	LOW	HIGH
MEAN	220.231	SUM	2863	75% Q3	234	95%	392	161	225
STD DEV	58.5721	VAR	3430.69	50% MED	221	90%	330	168	232
SKEW	2.21591	KURT	6.61979	25% Q1	174	10%	163	169	236
USS	671689	CSS	41168.3	0% MIN	161	5%	161	179	239
CV	26.5958	STD MEAN	16.245			1%	161	205	392

A 95% confidence interval for the mean yield is

- (a) 220.2 ± 1.96(58.6)      (b) 220.2 ± 1.96(16.2)  
 (c) 220.2 ± 2.18(58.6)  
 (d) 220.2 ± 2.18(16.2)      (e) 220.2 ± 2.16(16.2)

$$t^* = 2.18 \quad SE_{\bar{x}} = \frac{58.5721}{\sqrt{13}} = 16.2$$

8. The weights of 9 men have mean  $\bar{x}$  = 175 pounds and standard deviation  $s$  = 15 pounds. What is the standard error of the mean?

- (a) 58.3      (b) 19.4      (c) 5      (d) 1.7  
 (e) None of the above. The answer is \_\_\_\_\_.

$$Pop \geq 10(a) = 90$$

$$SE_{\bar{x}} = \frac{15}{\sqrt{9}} = 5$$

9. The Gallup Poll interviews 1600 people. Of these, 18% say that they jog regularly. The news report adds: "The poll had a margin of error of plus or minus three percentage points." You can safely conclude that

- (a) 95% of all Gallup Poll samples like this one give answers within ±3% of the true population value.  
 (b) The percent of the population who jog is certain to be between 15% and 21%.  
 (c) 95% of the population jog between 15% and 21% of the time.  
 (d) We can be 3% confident that the sample result is true.  
 (e) If Gallup took many samples, 95% of them would find that exactly 18% of the people in the sample jog.

**Part 2: Free Response** Communicate your thinking clearly and completely.

10. A steel mill's milling machine produces steel rods that are supposed to be 5 cm in diameter. When the machine is in statistical control, the rod diameters vary according to a Normal distribution with mean  $\mu = 5$  cm. A large sample of 150 rods produced by the machine yields a mean diameter of 5.005 cm and a standard deviation of 0.02 cm.

(a) Construct and interpret a 99% confidence interval for the true mean diameter of the rods produced by the milling machine.

T interval with 149 df.

Random  $\rightarrow$  NOT stated so may not be able to generalize findings.  
Independence  $\rightarrow$  assume Pop  $\geq 10(150)$   
 $= 1500$  steel rods

Large counts  $\rightarrow$  given the Pop distribution is normal so the sampling distribution is approximately normal.

$$\bar{x} \pm t^* \left( \frac{s}{\sqrt{n}} \right)$$

$\swarrow$  from calculator

$$5.005 \pm 2.609 \left( \frac{0.02}{\sqrt{150}} \right)$$
$$(5.001, 5.009)$$

We are 99% confident that the true mean diameter of all steel rods produced on this machine is between 5.001 cm and 5.009 cm.

(b) Does the interval in part a give you reason to suspect that the machine is not producing rods of the correct diameter? Explain your reasoning.

Yes. Since  $\mu = 5$  is not in our interval, it appears the machine is not working properly. Specifically, since the interval contains values all above 5, the machine is producing steel rods with larger diameters.

11. A survey of a random sample of 1280 student loan borrowers found that 448 had loans totaling more than \$20,000 for their undergraduate education.

(a) Construct and interpret a 90% confidence interval for the population proportion  $p$ .

1 prop z interval

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Random  $\rightarrow$  given as a random sample

Independence  $\rightarrow$  pop of student loan borrowers  $\geq 10(1280) = 12800 \checkmark$

$$\hat{p} = \frac{448}{1280} = .35$$

Large counts  $\rightarrow np = 448 \geq 10 \checkmark$   
and  $n(1-p) = 1280 - 448 = 832 \geq 10 \checkmark$

$$.35 \pm 1.645 \sqrt{\frac{.35(.65)}{1280}}$$

$$(.3281, .3719)$$

We are 90% confident that the true proportion of all student loan borrowers who owe more than \$20,000 is between .3281 and .3719.

(b) Students reported the total amount of loans they had obtained for their undergraduate education. No attempt was made to verify the loan amounts reported by students. How might this information affect your interpretation of the result from (a)?

Since the loan amounts were self-reported, they may not be accurate (response bias). This would change the values in the confidence interval.

(c) If you used this sample to construct a confidence interval for the mean amount of students' loans, could the resulting interval contain \$20,000? Justify your answer.

Since the proportion of borrowers who owed more than \$20,000 is 35%, which is quite a large amount, it seems possible that a confidence interval for the mean amount owed could include \$20,000. It really depends on the values of  $\bar{x}$  and  $s$ .