Name ______________

Date ______________

Prove each identity

① $\cos x\left(\tan x + \sin x \cot x\right) = \sin x + \cos^2 x$

② $\dfrac{(1-\cos x)(1+\cos x)}{\cos^2 x} = \tan^2 x$

③ $\dfrac{\sec^2 x - 1}{\sin x} = \dfrac{\sin x}{1-\sin^2 x}$

④ $\tan x + \sec x = \dfrac{\cos x}{1-\sin x}$

Answers

① $\cos x\left(\tan x + \sin x \cot x\right)$

$= \cos x \tan x + \cos x \sin x \cot x = \left(\dfrac{\cancel{\cos x}}{1}\right)\left(\dfrac{\sin x}{\cancel{\cos x}}\right) + \left(\dfrac{\cos x}{1}\right)\left(\dfrac{\cancel{\sin x}}{1}\right)\left(\dfrac{\cos x}{\cancel{\sin x}}\right)$

$= \sin x + \cos^2 x.$

② $\dfrac{(1-\cos x)(1+\cos x)}{\cos^2 x} = \dfrac{1-\cos^2 x}{\cos^2 x} = \dfrac{\sin^2 x}{\cos^2 x} = \tan^2 x$

(3) $$\frac{\sec^2 x - 1}{\sin x} = \frac{\tan^2 x}{\sin x} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\sin x}{1}} = \frac{\sin^{\cancel{2}} x}{\cos^2 x \cancel{\sin x}}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{\sin x}{1 - \sin^2 x}$$

(4) $$\tan x + \sec x = \frac{\sin x}{\cos x} + \frac{1}{\cos x} = \frac{\sin x + 1}{\cos x}\left(\frac{\cos x}{\cos x}\right)$$

$$= \frac{\cos x(\sin x + 1)}{\cos^2 x} = \frac{\cos x(\sin x + 1)}{1 - \sin^2 x} = \frac{\cos x \cancel{(\sin x + 1)}}{(1 - \sin x)\cancel{(1 + \sin x)}}$$

$$= \frac{\cos x}{1 - \sin x}$$