

Part 1: Multiple Choice. Circle the letter corresponding to the best answer

1. DDT is an insecticide that accumulates up the food chain. Predator birds can be contaminated with quite high levels of the chemical by eating many lightly contaminated prey. It is believed that this causes eggshells to be thinner and weaker than normal and makes the eggs more prone to breakage. An experiment was conducted where 16 sparrow hawks were fed a mixture of 3 ppm dieldrin and 15 ppm DDT (a combination often found in contaminated prey). The first egg laid by each bird was measured, and the mean shell thickness was found to be 0.19 mm. A "normal" eggshell has a mean thickness of 0.2 mm.

The null and alternative hypotheses are

- (a) $H_0 : \mu = 0.2; H_a : \mu < 0.2$
- (b) $H_0 : \mu < 0.2; H_a : \mu = 0.2$
- (c) $H_0 : \bar{x} = 0.2; H_a : \bar{x} < 0.2$
- (d) $H_0 : \bar{x} = 0.19; H_a : \bar{x} = 0.2$
- (e) $H_0 : \mu = 0.2; H_a : \mu \neq 0.2$

2 (tcdf(.8, 10000, 79))

2. In a test of $H_0 : \mu = 100$ against $H_a : \mu \neq 100$, a sample of size 80 produces $t = 0.8$ for the value of the test statistic. The P -value of the test is thus equal to

- (a) 0.20
- (b) 0.40
- (c) 0.29
- (d) 0.43
- (e) 0.21

3. A certain population follows a Normal distribution. You collect data and test the hypotheses

$H_0 : \mu = 1, H_a : \mu \neq 1$

You obtain a P -value of 0.022. Which of the following is true?

- (a) A 95% confidence interval for μ will include the value 1. $\rightarrow \alpha = .05 \rightarrow$ reject H_0
- (b) A 95% confidence interval for μ will include the value 0. $\rightarrow \alpha = .05 \rightarrow$ reject H_0
- (c) A 99% confidence interval for μ will include the value 1. $\rightarrow \alpha = .01 \rightarrow$ Fail 2 reject H_0
- (d) A 99% confidence interval for μ will include the value 0. $\rightarrow \alpha = .01 \rightarrow$ Fail 2 reject H_0
- (e) None of these is necessarily true.

4. Vigorous exercise helps people live several years longer (on the average). Whether mild activities like slow walking extend life is not clear. Suppose that the added life expectancy from regular slow walking is just 2 months. A statistical test is more likely to find a significant increase in mean life if

- (a) it is based on a very large random sample and a 5% significance level is used.
- (b) it is based on a very large random sample and a 1% significance level is used.
- (c) it is based on a very small random sample and a 5% significance level is used.
- (d) it is based on a very small random sample and a 1% significance level is used.
- (e) The size of the sample doesn't have any effect on the significance of the test.

5. In order to study the amounts owed to a particular city, a city clerk takes a random sample of 16 files from a cabinet containing a large number of delinquent accounts and finds the average amount owed to the city to be \$230 with a sample standard deviation of \$36. It has been claimed that the true mean amount owed on accounts of this type is greater than \$250. If it is appropriate to assume that the amount owed is a Normally distributed random variable, the value of the test statistic appropriate for testing the claim is

- (a) -3.33
- (b) -1.96
- (c) -2.22
- (d) -0.55
- (e) -2.1314

$\frac{230 - 250}{36/\sqrt{16}}$

6. Which of the following 95% confidence intervals would lead us to reject $H_0 : p = 0.30$ in favor of $H_a : p \neq 0.30$ at the 5% significance level?

- (a) (0.30, 0.38)
- (b) (0.19, 0.27)
- (c) (0.27, 0.31)
- (d) (0.24, 0.30)
- (e) None of these

↑ does not include .3

7. Bags of a certain brand of tortilla chips claim to have a net weight of 14 ounces. Net weights actually vary slightly from bag to bag and are Normally distributed with mean μ . A representative of a consumer advocate group wishes to see if there is any evidence that the mean net weight is less than advertised and so intends to test the hypotheses

$$H_0: \mu = 14 \text{ versus } H_a: \mu < 14$$

A Type I error in this situation would mean *Reject H_0 when H_0 is True*

- (a) concluding that the bags are being underfilled when they actually aren't.
 (b) concluding that the bags are being underfilled when they actually are.
 (c) concluding that the bags are not being underfilled when they actually are.
 (d) concluding that the bags are not being underfilled when they actually aren't.
 (e) none of these

8. An inspector inspects large truckloads of potatoes to determine the proportion p in the shipment with major defects prior to using the potatoes to make potato chips. Unless there is clear evidence that this proportion is less than 0.10, she will reject the shipment. To reach a decision she will test the hypotheses

$$H_0: p = 0.10, H_a: p < 0.10$$

using the large-sample test for a population proportion. To do so, she selects an SRS of 50 potatoes from the more than 2000 potatoes on the truck. Suppose that only two of the potatoes sampled are found to have major defects.

$$np_0 = 50(.1) = 5 \times$$

Which of the following conditions for inference about a proportion using a hypothesis test are violated?

- (a) The data are an SRS from the population of interest.
 (b) The population is at least 10 times as large as the sample.
 (c) n is so large that both np_0 and $n(1 - p_0)$ are 10 or more, where p_0 is the proportion with major defects if the null hypothesis is true.
 (d) There appear to be no violations.
 (e) More than one condition is violated.

9. What is the value of t^* , the critical value of the t distribution with 8 degrees of freedom, which satisfies the condition that the probability is 0.10 of being larger than t^* ?

- (a) 1.415 (b) 1.397 (c) 1.645 (d) 2.896

InvT(.9, 8)



10. The water diet requires one to drink two cups of water every half hour from when one gets up until one goes to bed, but otherwise allows one to eat whatever one likes. Four adult volunteers agree to test the diet. They are weighed prior to beginning the diet and after six weeks on the diet. The weights (in pounds) are

Person	1	2	3	4
Weight before the diet	180	125	240	150
Weight after six weeks	170	130	215	152

For the population of all adults, assume that the weight loss after six weeks on the diet (weight before beginning the diet – weight after six weeks on the diet) is Normally distributed with mean μ . To determine if the diet leads to weight loss, we test the hypotheses

$$H_0: \mu = 0, H_a: \mu > 0$$

$$p \text{ value} = 0.19$$

Based on these data we conclude that

- (a) we would not reject H_0 at significance level 0.10.
 (b) we would reject H_0 at significance level 0.10 but not at 0.05.
 (c) we would reject H_0 at significance level 0.05 but not at 0.01.
 (d) we would reject H_0 at significance level 0.01.
 (e) the sample size is too small to allow use of the t procedures.

Part 2: Free Response*Communicate your thinking clearly and completely.*

11. It is believed that the average amount of money spent per U.S. household per week on food is about \$98. A random sample of 100 households in a certain affluent community yields a mean weekly food budget of \$100 and a standard deviation of \$10. We want to test the hypothesis that the mean weekly food budget for all households in this community is higher than the national average.

- (a) Perform a significance test at the $\alpha = 0.05$ significance level.

T test with 99 degrees of freedom

$$H_0: \mu = 98$$

$$H_a: \mu > 98$$

$$t = \frac{100 - 98}{10/\sqrt{100}} = 2$$

Random: given

Independence: $Pop \geq 10(100) = 1000$

Normal: by CLT since $100 \geq 30$

$$p\text{-value} = .024$$

Since the p-value of .024 is below the significance level of .05, we have enough evidence that the mean weekly food budget for households in this community is greater than \$98.

- (b) Describe a Type I error in the context of this problem. What is the probability of making a Type I error?

We conclude that the mean weekly budget is more than \$98 when it isn't.

$$P(\text{Type I error}) = \alpha = .05$$

- (c) Describe a Type II error in the context of this problem. Give two ways to reduce the probability of a Type II error.

We conclude that the mean weekly budget is not more than \$98 when it actually is more than \$98.

Two ways to reduce the probability of a Type II error are:

- increase sample size n
- increase significance level α .

12. A growing number of employers are trying to hold down the costs that they pay for medical insurance for their employees. As part of this effort, many medical insurance companies are now requiring clients to use generic brand medicines when filling prescriptions. An independent consumer advocacy group wanted to determine if there was a difference, in milligrams, in the amount of active ingredient between a certain "name brand" drug and its generic counterpart. Pharmacies may store drugs under different conditions. Therefore, the consumer group randomly selected ten different pharmacies in a large city and filled two prescriptions at each of the pharmacies, one for the "name" brand and the other for the generic brand of the drug. The consumer group's lab then tested a randomly selected pill from each prescription to determine the amount of active ingredient in the pill. The results are given in the following table.

Pharmacy	1	2	3	4	5	6	7	8	9	10
Name Brand	245	244	240	250	243	246	246	246	247	250
Generic Brand	246	240	235	237	243	239	241	238	238	234

Based on these results, what should the consumer group's lab report about the difference in the active ingredient in the two brands of pills? Give appropriate statistical evidence to support your response.

paired t test with 9 degrees of freedom

μ_{diff} = difference in amount of active ingredient between name brand and generic pills (NB-gen)

$$H_0: \mu_{diff} = 0$$


$$\bar{X} = 6.6$$

$$H_a: \mu_{diff} \neq 0$$

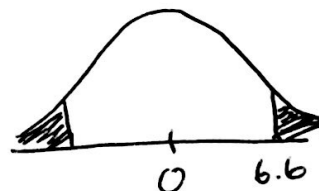
$$S_x = 5.275$$

Random \rightarrow well designed randomized selection of pills

Ind \rightarrow Pop of pills ≥ 10 (10) = 100

Normal \rightarrow Boxplot 
no outliers or strong skewness

$$t = \frac{6.6 - 0}{5.275 / \sqrt{10}} = 3.957$$



p-value = .0033

Since the p-value of .0033 is lower than any reasonable significance level, we have enough evidence that name brand drugs have more active ingredients than the generic counterpart.

13. Mars Inc., makers of M&M's candies, claims that they produce plain M&M's with the following distribution

Brown: 30%	Red: 20%	Yellow: 20%
Orange: 10%	Green: 10%	Blue: 10%

A bag of plain M&M's was selected randomly from the grocery store shelf, and the color counts were as follows:

Brown: 16	Red: 11	Yellow: 19	$n = 61$
Orange: 5	Green: 7	Blue: 3	

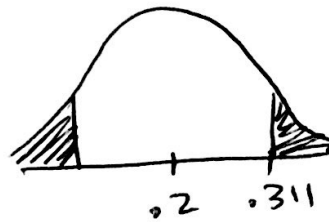
(a) Conduct an appropriate test of the manufacturer's claim for the proportion of yellow M&M's.

1 prop z test with $\alpha = .05$

$x = 19$
 $\hat{p} = \frac{19}{61} = .311$

$H_0: p = .2$

$H_a: p \neq .2$



Random \rightarrow bag randomly selected

Ind \rightarrow Pop. of M&M's $\geq 10(61) = 610$

Normal $\rightarrow np_0 = 61(.2) = 12.2 \geq 10$

$n(1-p_0) = 61(.8) = 48.8 \geq 10$

$$z = \frac{.311 - .2}{\sqrt{\frac{.2(.8)}{61}}} = 2.177$$

p-value = .029

Since p-value of .029 is below the significance level of .05, we have enough evidence that the proportion of yellow M&M's is higher than 20%.

(b) Based on this sample, construct and interpret a 90% confidence interval for the proportion of yellow M&M's candies produced by Mars.

check

Normal: $n\hat{p} = 19 \geq 10$

$n(1-\hat{p}) = 61-19 = 42 \geq 10$

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$.311 \pm 1.645 \sqrt{\frac{.311(.689)}{61}} \Rightarrow (.214, .409)$$

We are 90% confident that the true proportion of yellow M&M's in a bag of 61 candies is between 21.4% and 40.9%.