A Horse Race Expected Value

Name ______________________________

Period_____ Date _______________

Thanks to you, the betting office has realized some horses are more likely to win, so they will pay betters less if those horses win. (Way to go, party pooper!) Here are the new payouts:

Horse #	1	2	3	4	5	6	7	8	9	10	11	12
Payout	$200	$70	$35	$22	$16	$14	$10	$14	$16	$22	$35	$70

1. If you bet on horse 4, the *theoretical* probability you will win is _____ (refer to **#10** on the other sheet).

a. So there is a _____ probability you'll win $22, and a _____ probability you'll win $0. What are your expected earnings if you bet on horse 4?

b. But of course it costs money to place a bet! At this racetrack, it costs $2 to place a bet on any horse. *Now* what are your expected earnings/losses if you bet on horse 4?

2. Calculate the expected earnings/losses on a $2 bet for each horse. (You just did it for horse 4.) Refer to **#10** on the other sheet for P(win).

Horse #	1	2	3	4	5	6	7	8	9	10	11	12
P(win) **#10** on other sheet												
Winning Payout	$200	$70	$35	$22	$16	$14	$10	$14	$16	$22	$35	$70
P(lose)												
Losing Payout	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0
Exp. Val. if you bet on this horse												

(Remember, it costs $2 to place a bet!)

3. If you were going to bet on this race a million times (always picking the same horse)…

a. Which horse is the best (or least bad) choice? There may be more than one.

b. Which horse is the worst choice? There may be more than one.

c. Should you go through with your plan to bet a million times? Explain *mathematically* in two or three full sentences.

More Practice:

4. You have a choice of investing in two projects:
   Project A will result in a loss of $26,000 with a probability of .30, breaking even with a probability of .50, or a gain of $68,000 with a probability of .20.
   Project B will result in a loss of $71,000 with a probability of .20, breaking even with a probability of .65, or a gain of $143,000 with a probability of .15.
   Which investment is better? Justify your answer.

5. A men's soccer team plays soccer 0, 1, or 2 days a week. The probability that they play 0 days is 0.2, the probability that they play 1 day is 0.5, and the probability that they play 2 days is 0.3. Find the long-term average, μ, or expected value of the days per week the men's soccer team plays soccer.

6. Suppose you play a game of chance in which you choose 5 numbers from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. You may choose a number more than once. You pay $2 to play and could profit $100,000 if you match all 5 numbers in order (you get your $2 back plus $100,000). Over the long term, what is your *expected* profit of playing the game?

7. Suppose you play a game with a biased coin. You play each game by tossing the coin once. P(heads) = 2/3 and P(tails)= 1/3. If you toss a head, you pay $6. If you toss a tail, you win $10. If you play this game many times, will you come out ahead?

8. The Green Mountain Lottery in Vermont allows you to play a three digit number (0 to 9) and repeats are allowed. If you win, the prize is $500. What is the expected value of your winnings?

9. Suppose a fair coin is tossed three times and we let x= number of heads. Find $\mu = E(x)$.

A men's soccer team plays soccer 0, 1, or 2 days a week. The probability that they play 0 days is 0.2, the probability that they play 1 day is 0.5, and the probability that they play 2 days is 0.3. Find the long-term average, μ, or expected value of the days per week the men's soccer team plays soccer.

To do the problem, first let the random variable X = the number of days the men's soccer team plays soccer per week. X takes on the values 0, 1, 2. Construct a PDF table, adding a column xP(X=x). In this column, you will multiply each x value by its probability.

x	**P(X=x)**	***x*P(X=x)**
0	0.2	(0)(0.2) = 0
1	0.5	(1)(0.5) = 0.5
2	0.3	(2)(0.3) = 0.6

Table 1: **Expected Value Table**

This table is called an expected value table. The table helps you calculate the expected value or long-term average.

Add the last column to find the long term average or expected value:
(0)(0.2)+(1)(0.5)+(2)(0.3)= 0 + 0.5. 0.6 = 1.1.

The expected value is 1.1. The men's soccer team would, on the average, expect to play soccer 1.1 days per week. The number 1.1 is the long term average or expected value if the men's soccer team plays soccer week after week after week. We say μ=1.1

Suppose you play a game of chance in which you choose 5 numbers from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. You may choose a number more than once. You pay $2 to play and could profit $100,000 if you match all 5 numbers in order (you get your $2 back plus $100,000). Over the long term, what is your *expected* profit of playing the game?

To do this problem, set up an expected value table for the amount of money you can profit.

Let X = the amount of money you profit. The values of x are not 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Since you are interested in your profit (or loss), the values of x are 100,000 dollars and -2 dollars.

To win, you must get all 5 numbers correct, in order. The probability of choosing one correct number is

$\frac{1}{10}$

because there are 10 numbers. You may choose a number more than once. The probability of choosing all 5 numbers correctly and in order is:

$\frac{1}{10}$

*

$\frac{1}{10} * \frac{1}{10} * \frac{1}{10} * \frac{1}{10} * = 1 * 10^{-5} = 0.00001$ (1)

Therefore, the probability of winning is 0.00001 and the probability of losing is

$1 - 0.00001 = 0.99999$ (2)

The expected value table is as follows.

	x	P(X=x)	xP(X=x)
Loss	-2	0.99999	(-2)(0.99999)=-1.99998
Profit	100,000	0.00001	(100000)(0.00001)=1

Table 3: Add the last column. -1.99998 + 1 = -0.99998

Since -0.99998 is about -1, you would, on the average, expect to lose approximately one dollar for each game you play. However, each time you play, you either lose $2 or profit $100,000. The $1 is the average or expected LOSS per game after playing this game over and over.

	x	$P(X=x)$	$xP(X=x)$
WIN	10	$\frac{1}{3}$	$\frac{10}{3}$
LOSE	-6	$\frac{2}{3}$	$\frac{-12}{3}$

Table 5

[Hide Solution]

Problem 3

What is the expected value, μ? Do you come out ahead?

Solution

Add the last column of the table. The expected value μ=

$\frac{-2}{3}$. You lose, on average, about 67 cents each time you play the game so you do not come out ahead.