

Multiple Choice

1. A basketball player makes 70% of her free throws. She takes 7 free throws in a game. If the shots are independent of each other, the probability that she makes 5 out of the 7 shots is about

- (a) 0.635.
 (b) 0.318.
 (c) 0.015.
 (d) 0.329.
 (e) 0.245.

$${}^7C_5 (.7)^5 (.3)^2 \quad n=7 \quad p=.7$$

$$k=5$$

binom PDF (7, .7, 5)

2. It has been estimated that as many as 70% of the fish caught in certain areas of the Great Lakes have liver cancer due to the pollutants present. A sample of 130 fish is caught and inspected for signs of liver cancer. The number of infected fish within two standard deviations of the mean is

- (a) (81, 101).
 (b) (86, 97).
 (c) (63, 119).
 (d) (36, 146).
 (e) (75, 107).

$$n=130 \quad p=.7$$

$$\mu = 130(.7) = 91$$

$$\sigma = \sqrt{130(.7)(.3)} = 5.22$$

$$\mu + 2\sigma \approx 101 \quad \mu - 2\sigma \approx 81$$

3. In a triangle test a tester is presented with three food samples, two of which are alike, and is asked to pick out the odd one by tasting. If a tester has no well-developed sense of taste and can pick the odd one only by chance, what is the probability that in five trials he will make four or more correct decisions?

- (a) 0.045
 (b) 0.004
 (c) 0.041
 (d) 0.959
 (e) 0.955

$$n=5 \quad p=1/3$$

$$P(X \geq 4) = 1 - P(X \leq 3)$$

1 - binom CDF (5, 1/3, 3)

4. A set of 10 cards consists of 5 red cards and 5 black cards. The cards are shuffled thoroughly and you turn cards over, one at a time, beginning with the top card. Let X be the number of cards you turn over until you observe the first red card. The random variable X has which of the following probability distributions?

- (a) The Normal distribution with mean 5
 (b) The binomial distribution with $p = 0.5$
 (c) The geometric distribution with probability of success 0.5
 (d) The uniform distribution that takes value 1 on the interval from 0 to 1
 (e) None of the above

trials not independent
 p not constant

5. Seventeen people have been exposed to a particular disease. Each one independently has a 40% chance of contracting the disease. A hospital has the capacity to handle 10 cases of the disease. What is the probability that the hospital's capacity will be exceeded?

- (a) 0.965
 (b) 0.035
 (c) 0.989
 (d) 0.011
 (e) 0.736

$$n = 17$$

$$p = 0.4$$

$$P(X > 10) = 1 - P(X \leq 10)$$

1 - binom CDF (17, 0.4, 10)

6. Refer to the previous problem. Planners need to have enough beds available to handle a proportion of all outbreaks. Suppose a typical outbreak has 100 people exposed, each with a 40% chance of coming down with the disease. Which is **not correct**?

- (a) This scenario satisfies the assumptions of a binomial distribution.
 (b) About 95% of the time, between 30 and 50 people will contract the disease.
 (c) Almost all of the time, between 25 and 55 people will contract the disease.
 (d) On average, about 40 people will contract the disease.
 (e) Almost all of the time, less than 40 people will be infected.

$$n = 100$$

$$\mu = 40$$

$$p = 0.4$$

$$\sigma = 4.89$$

$$\left. \begin{array}{l} np \geq 10 \checkmark \\ n(1-p) \geq 10 \checkmark \end{array} \right\} \text{approx Normal}$$

7. There are 10 patients on the neonatal ward of a local hospital who are monitored by 2 staff members. If the probability of a patient requiring emergency attention by a staff member is 0.3, what is the probability that there will not be sufficient staff to attend all emergencies? Assume that emergencies occur independently.

- (a) 0.3828
 (b) 0.3000
 (c) 0.0900
 (d) 0.9100
 (e) 0.6172

$$p = 0.3 \quad q = 0.7$$

$$P(X > 2)$$

$$= 1 - P(X \leq 2)$$

$$1 - \text{binomCDF}(10, 0.3, 2)$$

8. In 1989 *Newsweek* reported that 60% of young children have blood lead levels that could impair their neurological development. Assuming that a class in a school is a random sample from the population of all children at risk, the probability that more than 3 children have to be tested until one is found to have a blood level that may impair development is

- (a) 0.064.
 (b) 0.096.
 (c) 0.64.
 (d) 0.16.
 (e) 0.88.

$$\text{geometric CDF} \quad p = 0.6 \quad q = 0.4$$

$$P(X > 3) = q^n = 0.4^3 = 0.064$$

Free Response. Answer in clear and concise sentences and show your calculations where appropriate.

9. Would most wives marry the same man again, if given the chance? According to a poll of 608 married women conducted by *Ladies Home Journal* (June 1988), 80% would, in fact, marry their current husbands. Assume that the women in the sample were randomly selected from among all married women in the United States. Does the number X in the sample who would marry their husbands again have a binomial probability distribution? Explain.

Yes because it meets the four criteria:

① 2 possible outcomes (same man or not)

③ $n = 608$
so set # of trials

② independent trials (sampled randomly)

④ $p = 80\%$

10. About 50% of male Internet users 18-34 visit an auction site such as eBay at least once a month. Interview a random sample of 12 male Internet users aged 18 to 34.

- (a) What is the distribution of the number of males who have visited an online auction site in the past month?

$$B(n, p) \rightarrow B(12, .5)$$

- (b) If you interview 12 at random, what is the mean of the count X who have visited an auction site?

$$\mu = np = 12(.5) = 6$$

- (c) What is the probability that at least 8 of the 12 have visited an auction site in the past month?

$$P(X \geq 8) = {}_{12}C_8 (.5)^8 (.5)^4 + {}_{12}C_9 (.5)^9 (.5)^3 + \dots \\ + {}_{12}C_{12} (.5)^{12} (.5)^0 = .1938$$

1 - binom CDF (12, .5, 7)

- (d) Repeat the calculations in (b) for samples of size 120 and 1200. What happens to the mean count of successes as the sample size increases?

μ increases as n increases

$$n = 120 \rightarrow \mu = 60$$

$$n = 1200 \rightarrow \mu = 600$$

11. Suppose that an elite, small, selective college admits, on average, 1 of every 7 males that applies. Let X be the number of male applicants that are admitted.

(a) Show that the conditions for a geometric setting are satisfied. If you need to assume that one or more of these conditions are satisfied, state this.

- ① $p = 1/7$
- ② applicants' admission can be assumed to be independent
- ③ n is not a set #
- ④ there are 2 outcomes - accepted / not accepted

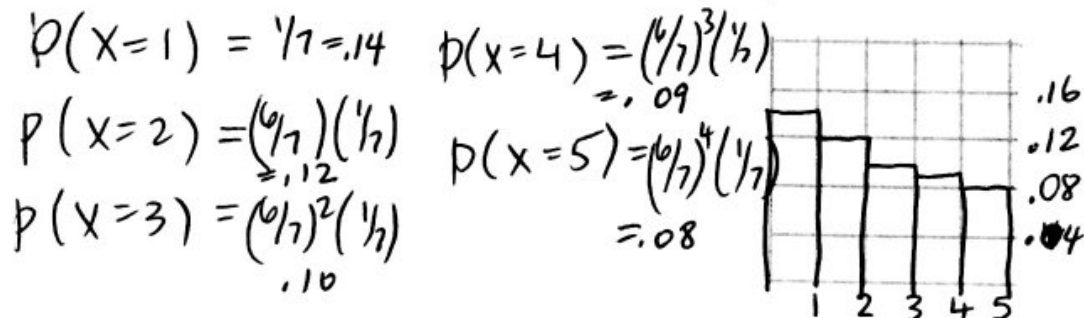
(b) What are the mean and standard deviation of X ? Interpret the mean in the context of this problem.

$$\mu = \frac{1}{p} = \frac{1}{1/7} = 7$$

On average, it will take 7 students for one to be admitted.

$$\sigma = \sqrt{\frac{1-p}{p^2}} = \sqrt{\frac{6/7}{(1/7)^2}} = \sqrt{42} = 6.48$$

(c) Construct a probability distribution table and histogram, out to $n = 5$.



(d) What is the probability that the Admissions Committee has to look at exactly 5 applicants until it finds a male student to admit? At most 5 applicants?

$$P(X=5) = \left(\frac{6}{7}\right)^4 \left(\frac{1}{7}\right) = .077$$

$$P(X \leq 5) = 1 - \left(\frac{6}{7}\right)^5 = .537$$

Geometric CDF (p, n)