AP Statistics— Chapter 6.3
Binomial Distribution Practice

Name Key

1. **Lefties.** Assume that 13% of people are left handed. If we select 5 people at random, find the probability:

(a) There are exactly 3 lefties in the group.

$P(x=3)=\binom{5}{3}(0.13)^3(.87)^2=\underline{.0166}$

(b) There are at least 3 lefties in the group.

$P(x\geq 3)=1-P(x\leq 2)=1-\left[\binom{5}{0}(.13)^0(.87)^5+\ldots+\binom{5}{2}(.13)^2(.87)^3\right]=1-.982$
$=\underline{.0179}$

(c) There are no more than 3 lefties in the group.

$P(x\leq 3)=\binom{5}{0}(.13)^0(.87)^5+\ldots+\binom{5}{3}(.13)^3(.87)^2=\underline{.9987}$

2. **More Lefties.** Suppose we choose 12 people instead of 5 in problem 1.

(a) What's the probability that they're not all right-handed?

$P(x\geq 1)=1-P(x=0)=1-\binom{12}{0}(.13)^0(.87)^{12}=\underline{.812}$

(b) What's the probability that there are no more than 10 righties? → lets call Y = # of righties

$P(Y\leq 10)=\binom{12}{0}(.87)^0(.13)^{12}+\ldots+\binom{12}{10}(.87)^{10}(.13)^2=\underline{.475}$

(c) What's the probability that there are exactly 6 of each?

$P(x=6)=\binom{12}{6}(.13)^6(.87)^6=\underline{.0019}$

↳ indirectly says Y=6 also!

(d) What's the probability that the majority are right-handed?

$P(Y\geq 7)=P(x\leq 5)=\binom{12}{0}(.13)^0(.87)^{12}+\ldots+\binom{12}{5}(.13)^5(.87)^7=\underline{.998}$

3. **Arrows.** An Olympic archer is able to hit the bull's-eye 80% of the time. Assume each shot if independent of the others. If she shoots 6 arrows, what's the probability that:

$P(HIT)=.8$
$P(MISS)=.2$

(a) She misses the bull's-eye at least once.

$P(M\geq 1)=1-P(M=0)=1-\binom{6}{0}(.2)^0(.8)^6=\underline{.737}$

(b) She gets exactly 4 bull's-eyes.

$P(H=4)=\binom{6}{4}(.8)^4(.2)^2=\underline{.246}$

(c) She gets at least 4 bull's-eyes.

$P(H\geq 4)=\binom{6}{4}(.8)^4(.2)^2+\ldots+\binom{6}{6}(.8)^6(.2)^0=\underline{.901}$

(d) She gets at most 4 bull's-eyes.

$P(H\leq 4)=\binom{6}{0}(.8)^0(.2)^6+\ldots+\binom{6}{4}(.8)^4(.2)^2=\underline{.345}$

4. More Arrows. Suppose our archer from problem 3 shoots 10 arrows.

(a) What's the probability she never misses?

$P(M=0) = \binom{10}{0}(.2)^0(.8)^{10} = \underline{.107}$

(b) What's the probability that there are no more than 8 bulls-eyes?

$P(H \leq 8) = \binom{10}{0}(.8)^0(.2)^{10} + \ldots + \binom{10}{8}(.8)^8(.2)^2 = \underline{.624}$

(c) What's the probability that there are exactly 8 bull's-eyes?

$P(H=8) = \binom{10}{8}(.8)^8(.2)^2 = \underline{.302}$

(d) What's the probability that she hits the bull's-eyes more often than she misses?

$P(H \geq 6) = \binom{10}{6}(.8)^6(.2)^4 + \ldots + \binom{10}{10}(.8)^{10}(.2)^0 = \underline{.967}$

5. Tennis anyone? A certain tennis player makes a successful first serve 70% of the time. Assume each serve is independent. If she serves 6 times, what's the probability she gets: $P(S) = .7$ $P(F) = .3$

(a) all 6 first serves in?

$P(S=6) = \binom{6}{6}(.7)^6(.3)^0 = \underline{.118}$

(b) exactly 4 first serves in?

$P(S=4) = \binom{6}{4}(.7)^4(.3)^2 = \underline{.324}$

(c) at least 4 serves in?

$P(S \geq 4) = \binom{6}{4}(.7)^4(.3)^2 + \ldots + \binom{6}{6}(.7)^6(.3)^0 = \underline{.744}$

(d) no more than 4 serves in?

$P(S \leq 4) = \binom{6}{0}(.7)^0(.3)^6 + \ldots + \binom{6}{4}(.7)^4(.3)^2 = \underline{.58}$

6. Frogs. A wildlife biologist examines frogs for a genetic trait he suspects may be linked to sensitivity to industrial toxin in the environment. Previous research has found that this trait is usually found in 1 of every 8 frogs. He collects and examines a dozen frogs. What's the probability he finds the trait in:

$P(T) = 1/8$

(a) none of the 12 frogs?

$P(T=0) = \binom{12}{0}\left(\frac{1}{8}\right)^0(7/8)^{12} = \underline{.201}$

(b) at least 2 frogs?

$P(T \geq 2) = \binom{12}{2}\left(\frac{1}{8}\right)^2\left(\frac{7}{8}\right)^{10} + \ldots + \binom{12}{12}\left(\frac{1}{8}\right)^{12}\left(\frac{7}{8}\right)^0 = \underline{.453}$

(c) 3 or 4 frogs?

$P(T = 3 \text{ or } 4) = P(T=3) + P(T=4) = \binom{12}{3}\left(\frac{1}{8}\right)^3\left(\frac{7}{8}\right)^9 + \binom{12}{4}\left(\frac{1}{8}\right)^4\left(\frac{7}{8}\right)^8 = \underline{.171}$

(d) no more than 4 frogs?

$P(T \leq 4) = \binom{12}{0}\left(\frac{1}{8}\right)^0\left(\frac{7}{8}\right)^{12} + \ldots + \binom{12}{4}\left(\frac{1}{8}\right)^4\left(\frac{7}{8}\right)^8 = \underline{.989}$