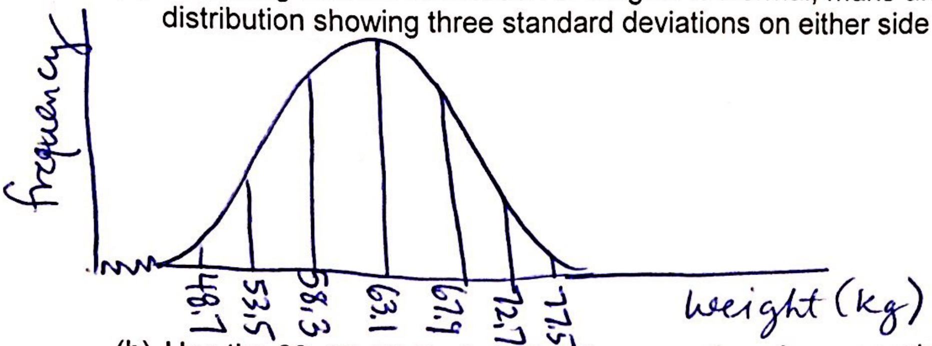
AP Statistics—Ch. 2
Normal Distributions Practice

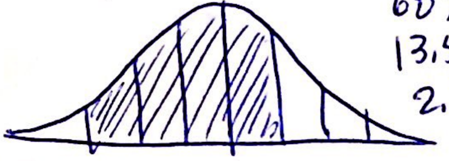
Name KEY

1. A study of elite distance runners found a mean body weight of 63.1 kilograms (kg), with a standard deviation of 4.8 kg.

(a) Assuming that the distribution of weights is Normal, make an accurate sketch of the weight distribution showing three standard deviations on either side of the mean.



(b) Use the 68–95–99.7 rule to find the proportion of runners whose body weight is between 48.7 and 67.9 kg. Show your method.

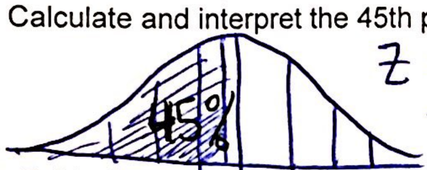


68% fall between 58.3 – 67.9
13.5% fall between 53.5 – 58.3
2.35% fall between 48.7 – 53.5

68 + 13.5 + 2.35 = 83.85

About 83.85% of runner body weights are in this range.

(c) Calculate and interpret the 45th percentile of the runners' body weight distribution.

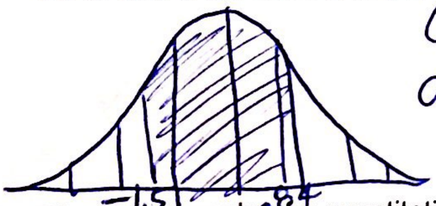


$z = -.126$

$-.126 = \frac{x - 63.1}{4.8} \rightarrow x = 62.495$

45% of runners have weights below 62.495 kg.

2. Use Table A or your calculator to find the proportion of observations from a standard Normal distribution that satisfies $-1.51 < Z < 0.84$. Sketch the Normal curve and shade the area under the curve that is the answer to the question.



area below $z = .84 \Rightarrow .8$

area below $z = -1.51 \Rightarrow .066$

$.8 - .066 = \boxed{.734}$

3. Give an example of a quantitative variable that does not have a Normal distribution. Justify your answer.

Age of Americans at death – should be skewed left since most people die at an older age but some die younger

4. The length of human pregnancies from conception to birth varies according to a distribution that is approximately Normal with mean 266 days and standard deviation 16 days. Use the 68–95–99.7 rule to answer the following questions. **Show your work.**

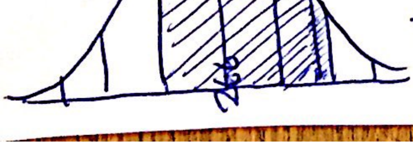
(a) How short are the shortest 2.5% of all pregnancies?

By Empirical Rule, these pregnancies are below $z = -2$

$-2 = \frac{x - 266}{16} \Rightarrow x = 234$

The shortest 2.5% of pregnancies are below 234 days

(b) What percent of pregnancies last between 250 and 298 days?



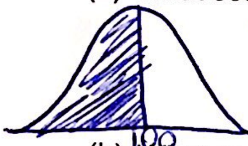
$z = \frac{298 - 266}{16} = 2$

$z = \frac{250 - 266}{16} = -1$

68% + 13.5% = 81.5% of pregnancies are between 250 & 298 days.

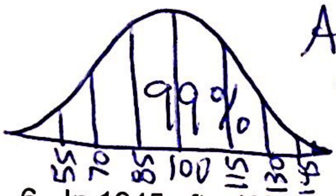
5. The scores of a reference population on the Wechsler Intelligence Scale for Children (WISC) are Normally distributed with μ = 100 and st. dev. = 15.

(a) What score would represent the 50th percentile? Explain.



The mean is the 50^{th} percentile since 50% of scores are below the mean. 100 on the WISC is the 50^{th} percentile.

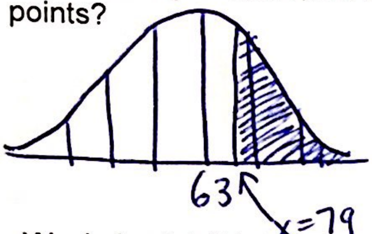
(b) What score would place a student in the top 1% of the scores? Show your method.



A z-score of 2.326 has 1% above and 99% below. $2.326 = \frac{X-100}{15} \rightarrow x = 133.54$

The top 1% of scores fall above 133.54 points.

6. In 1945 after WWII, all servicemen were given point scores based on length of service, number of purple hearts, number of campaigns, etc. Assuming that the distribution is N(63, 20), *how many* men from an army of 8,000,000 would be discharged if the army discharged all men with more than 79 points?

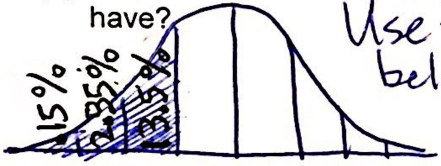


$Z = \frac{79-63}{20} = 0.8$

21.2% of men would be discharged, so (8,000,000)(.212) = 1,696,000 men

7. Wechsler Adult Intelligence Scale (WAIS) scores for young adults are N(110, 25).

(a) If someone's score were reported as the 16th percentile, about what score would that individual have?

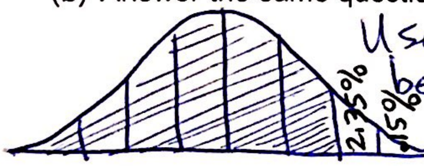


Use the Empirical Rule → bottom 16% is below $Z = -1$

$-1 = \frac{X-110}{25} \rightarrow x = 85$

The 16^{th} percentile on the WAIS is about 85.

(b) Answer the same question for the 97.5th percentile.

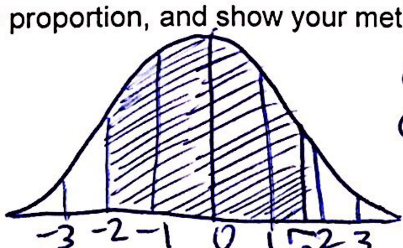


Use the Empirical Rule → bottom 97.5% is below $Z = 2$

$2 = \frac{X-110}{25} \rightarrow X = 160$

The 97.5 percentile on the WAIS is about 160.

8. Use Table A or your calculator to find the proportion of observations from a standard Normal distribution for which $-2 < Z < 1.67$. Sketch a standard Normal curve, shade the area representing the proportion, and show your method of calculating the proportion.



area below $Z = 1.67 \Rightarrow .953$

area below $Z = -2 \Rightarrow .023$

$.953 - .023 = \boxed{.93}$

9. The distribution of heights of adult American men is approximately Normal with mean 69 inches and standard deviation 2.5 inches.

(a) How tall must a man be to be in the shortest 5% of all adult men? Show your method.

$Z = -1.645$

$-1.645 = \frac{X-69}{2.5} \rightarrow X = 64.89$

He must be at most 64.9 inches tall.

(b) How tall must a man be to be in the tallest 10% of all adult men? Show your method.

$Z = 1.282$

$1.282 = \frac{X-69}{2.5} \rightarrow X = 72.21$

He must be at least 72.21 inches tall.