AP Statistics—Unit 7
Mean and Variance of Random Variables Practice

Name Key

1. The probabilities that a randomly selected customer purchases 1, 2, 3, 4, or 5 items at a convenience store are 0.32, 0.12, 0.23, 0.18, and 0.15, respectively.

(a) Construct a probability distribution (table) for the data, and verify that this is a legitimate probability distribution.

X	1	2	3	4	5
P(X)	.32	.12	.23	.18	.15

It is legitimate b/c all prob. sum to 1.

(b) Calculate μ_x. Interpret this value in the context of this problem.

$$\mu_x = 1(.32) + 2(.12) + \dots = 2.72$$

On average, people tend to buy 2.72 items.

(c) Find the standard deviation of X.

$$\sigma_x = \sqrt{(1-2.72)^2(.32) + (2-2.72)^2(.12) + \dots} = 1.45$$

(d) Suppose two customers, A and B, are selected at random. Find the mean and standard deviation of the difference in the number of items purchased by A and by B. Show your work.

$$\mu_{A-B} = \mu_A - \mu_B = 2.72 - 2.72 = 0$$

$$\sigma_{A-B} = \sqrt{\sigma_A^2 + \sigma_B^2} = \sqrt{1.45^2 + 1.45^2} = 2.05$$

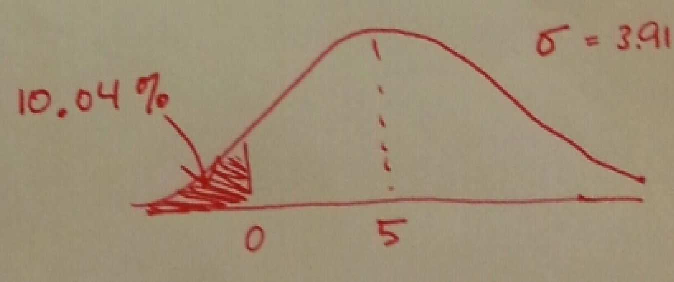
2. Suppose that the mean height of policemen is 70 inches with a standard deviation of 3 inches. And suppose that the mean height for policewomen is 65 inches with a standard deviation of 2.5 inches.

If heights of policemen and policewomen are Normally distributed, find the probability that a randomly selected policewoman is taller than a randomly selected policeman.

$$\mu_{M-W} = 70 - 65 = 5$$

$$\sigma_{M-W} = \sqrt{\sigma_M^2 + \sigma_W^2} = \sqrt{9 + 6.25} = 3.91$$

If the woman is taller than the man, that means μ_{M-W} would be negative.



$$z = \frac{0-5}{3.91} = -1.28$$

3. A single toss of a balanced coin has either 0 or 1 head, each with probability 1/2. What are the mean and standard deviation of the number of heads?

X	0	1
$P(X)$	1/2	1/2

$$\mu_x = 0(1/2) + 1(1/2) = 1/2$$

$$\sigma_x = \sqrt{(0-1/2)^2(1/2) + (1-1/2)^2(1/2)} = 1/2$$

4. Toss a coin four times. Use the rules for means and variances to find the mean and standard deviation of the total number of heads.

$$\mu_{total} = \mu_1 + \mu_2 + \mu_3 + \mu_4 = 1/2 + 1/2 + 1/2 + 1/2 = 2$$

$$\sigma_{total} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2} = \sqrt{1/4 + 1/4 + 1/4 + 1/4} = 1$$

5. Scores on the Mathematics part of the SAT college entrance exam in a recent year had mean 519 and standard deviation 115. Scores on the Verbal part of the SAT had mean 507 and standard deviation 111.

(a) What is the mean of the total SAT score (Math plus Verbal)?

$$\mu_{total} = \mu_m + \mu_v = 519 + 507 = 1026$$

(b) If you can calculate the standard deviation of the total SAT score, do it. If not, explain clearly why you can't.

To combine variances, they must be independent. In this case the same person is taking each part so the two parts are not independent.