## AP STATISTICS

# AP EXAM STUDY GUIDE

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**Additional Topics** 

BONUS Topic: Advice for the AP Exam (from someone who's passed six of them)

#### You are responsible for...

- Completing this study guide (5 points per topic)
- Completing the Practice Problems (5 points per topic)
- Studying hard and doing your best!

## **Topic 1:** Sampling Techniques and Sources of Bias (2.2)

- 1. Know and understand the difference between a *population* and *sample* 
  - taking, a measurement from every subject /object creates a population parametr, taking a measure ment from a subset.

    Why do we often measure samples instead of populations?

    Populations take to much time, may be impossible How is each one measured (what do we use to measure them)?
- 2. Know the different types of bias and how to spot them in different situations
  - Bias is anything that causes a sample to be not representative of the population of interest
    - You must be able to articulate <u>what</u> the bias is, <u>why</u> it should be considered bias, and <u>how</u> it distorts the results from what they otherwise might be.
  - What is the difference between sampling error and sampling bigs? Sampling error should be random, due to differences in subjects not design, sampling bias is approblem with the design.
  - How can a small sample size affect the validity of the sample? (this is related to sampling error rather than bias) Increasing sample size will not reduce beas it can reduce sampling error

Define the types of <b>sampling bias</b> (a bias in <i>who</i> was in the sample)	Define the types of <b>response bias</b> (a bias in <i>what</i> the sample is saying)
Mot every one who should be in the study is included in the sample	Cause the response to differ from the subjects true response.
Not everyone included in the Study has a recorded response	Measure ment devices that are mis calibrated or broken leads
Voluntary response bias  People in the study have a reason to be included, volunteer, ore not a part of a random process	to measurements that are different from true.

3. Know the different types of sampling techniques and how to identify which one is being used (as well as the advantages and disadvantages of each) Simple Random Sample (SRS) Stratified Random Sample When pop is easily divided on a veriable that may affect, the response, then take A sample taken in such a way that all samples of size a SRS from each subset n have an equal chance of \*Stratifying will reduce variability of possible sample results! When pop is divided into theterospross sub groups randonly select n sub groups and include overyone in the classic Systematic Random Sample Cluster Sample Pick a random # from I tom, then start there and every not person after that is included in the study Multistage Sample Convenience Sample. Sample taken with out any random process 4. Know how to design a random sampling procedure Random number generator will be your friend! "Describe a method..." (NOTE: blanks will be filled in with the context of the problem!) o START WITH: Assign each \_\_\_\_\_ (unit, subject, etc.) a different number between \_\_\_\_ and o Describe how you will implement the <u>sampling method</u> you want to use o Randomly select \_\_\_\_\_ numbers, ignoring repeats, and include the \_\_\_\_\_ (unit, subject, etc.) that corresponds with those numbers in your sample. **Example:** Mr. Frederick wants to create an advisory committee of 20 randomly-selected students out of the 1,950 students at Grant High School. Describe how he could do so using a... Systematic Random Sample

If the students from 1+97, Start with that

Select I number 6 to 1+97, Start with that Simple random sample # the students from 1 to 1950 use a RNG to generate 20 unique #'s student to be in the study then every anthe student after that will be included in from 1 to 1950, The students whose name corresponds to the # will be included in Stratified Random Sample of the shot sty 1 1950 Cluster Sample # the 97 honerouns for 1 to 97 use a RNG to get a number from I to 97.

All students in that room are to be included grade in school using a RNG select 5 students from each grade, those 20 stubits will be helided in the stuly in the studi Multistage Sample Survey the Rivst 20 students that enter the main doors

## 1. Know the vocabulary of experiments and experimental design

• What is the difference between an Experiment and an Observational Study? Which one lets us establish cause-and-effect relationships? *HINT:* There is one "dead giveaway" keyword when identifying an experiment. It starts with the letter A.

Treatments are assigned to subjects/objects

• Define Treatment –

Explanatory variable manipulated by the researcher

- Define Confounding variables that are not of interest to the shot of that may effect the response variable
- one new bur of a set of orbital subjects that are in Nally agricalent/smallest unit to which a treatment can be applied
- 2. Know the four principles of a good experiment
  - · Direct Control
  - · Blocking
  - · Randonitation
  - · Replication

## 3. Know methods for controlling an experiment to prevent bias

- Control group (what is it, and what does it allow us to do?)

  (NOTE: A control group is NOT mandatory; it is just one way to get comparison, which IS mandatory)

  allows the response be haves

  when the treatment is not used
- · Placebo effect—

  the a freatment with no active ingredients, used to

  Compare if the process of the freetment has an effect on the response

- Blind study The subject closs not know which treatment was recoved
- Double-blind study The subject and the person measuring the response do not know the treatment recire wed
- 4. Know the different types of experimental design and how to identify which one is being used (as well as the advantages and disadvantages of each)
  - Completely Randomized Design a design that uses randomization of factors to control the effect of extraneous variables
  - Randomized Block Design ("Blocking") treat ments are assigned within blocks of delpart subjects, each treatments are used in each block Used to control an extraneous variable that randomization Matched Pairs Design alone may not Subjects are grouped into pairs based on an extraneus variable them within each pair subjects are randomly assigned a treat ment
- 5. Be able to discuss generalizability the extent to which the results of a sample (or experimental group) can be applied to a certain population
  - You can generalize to the population from which the sample or experimental group was taken
  - BIAS can hurt (or even eliminate) generalizability. You need RANDOMNESS to avoid this!
    - o For example, a study that consists of volunteers should only be generalized to those volunteers! You might be able to generalize to "people who are similar to the volunteers," but absolutely no further, because they weren't randomly selected!
    - o NOTE: Even a relatively small sample size (not ridiculously small, but somewhat small) can be valid as long as it's random!

#### Example:

A researcher studied a random sample of 100 teens in Oklahoma. To which populations will the results of this researcher's findings be generalizable? (Circle ALL that apply)

A. The 100 Oklahoma teens in the study

All teens in Oklahoma

C. All teens no not all teens were in the sampling frame

D. All Oklahomans no not all Okl. were in Sampling frame

1. The 5 things you should discuss when analyzing a **distribution** of data:

**NOTE:** If asked to <u>compare</u> data sets, make sure you <u>explicitly</u> compare them (For example, "The first distribution has a greater mean than the second distribution, while the second distribution has a greater spread than the first")

#### 2. Center

Measure	How to find it	Resistant to the effects of outliers?
Mean Population: $\mu$ Sample: $\bar{x}$	$\frac{\sum x}{n}$	No
Median	(n/2 +,5) that we in an ordered data set	Yes

• The best one to use is usually mean, unless the data is skewed, at which point median should be used

3. Shape

Shape	Normal	Skewed Left	Skewed Right	Uniform	Bimodal
Sketch				1-1-	1
Which is greater, mean or median? (or are they = )	=	x < 92	x > q 2	x = 92	x = 92

## 4. Spread

Measure	Paired with (mean or median)	How to find it	Resistant to the effects of outliers?
Standard Deviation Population: $\sigma$ Sample: $s$	mean	$\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{n}}  s = \sqrt{\frac{\Sigma(x-\mu)^2}{n-1}}$ Or use 1-Var Stats!	n 0
Variance Population: σ² Sample: s²	mean	$\sigma^{2} = \frac{\Sigma(x-\mu)^{2}}{n}  S^{2} = \frac{\Sigma(x-\mu)^{2}}{n-1}$ Or use 1-Var Stats!	No
Lower Quartile (Q1)	a medren	Midpoint of Minimum and Median Or use 1-Var Stats!	yes
Upper Quartile (Q3)	median	Midpoint of Median and Maximum <i>Or use 1-Var Stats!</i>	yes

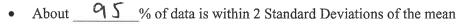
Range	/ \			
	either	max	-mn	no
Interquartile Range (IQR)	either med.	$a_3$	- Q,	yes
• What is an outlier?  Any vulve	more than	1.5TG	er away from	
of numbers to change				ddition of an outlier causes the position
How to identify outli	iers: IQR TEST (re	emember, this	s is a <i>general guideline</i> , n	ot a strict rule!)
How it works:				
owir < Q, - 1.5	ICR	Oner (one	< 32 - 1.5 (12)	ed = 36, Q3 = 44, Max = 51  vper > 44+1,5(Tax)  upper > 62
pper > Q3 + 1.5	IRR	an outlie	nt below 14 or a r. Outliers in this data so cst one lower	above 62 can be considered et:
Boxplot	Stemplot	-11 16	Dotplot	Histogram $\eta = 31$
10 20 30 40 50	6 9 7 8 7 9 0 10 0	8 8 9 6 7 7 8 means 68	0 1 2 3 4  Mandabilits  Numbers of Brothers and Sister	
	key to sho numbers:  Do not sho sho numbers:  Do not sho	cip stems back to aplot, ad stem leaf  Girls 1 268 3446689 436	So easy a caveman could do it!	<ul> <li>Notes:</li> <li>X-axis shows intervals, y-axis shows the frequency (number of data points that belong in that interval)</li> <li>Finding the median: Figure out how many data points there are, use n+1/2 to find the position of the median, then figure out which interval contains that position!</li> <li>EXAMPLE:  Number of data points:  3 1  Position of median:  Interval containing median:  75 4 80</li> </ul>

## **Topic 4:** Normal Distributions and Z-Scores (Notes: 4.4 and 7.6)

- 1. Know how to analyze a normal distribution
  - THEORETICAL distribution (in reality, we consider data to be \_\_approx i mately
  - It's like a histogram in which the center is the and the intervals are each one



- 2. Know how to use the **Empirical Rule** 
  - About 6 % of data is within 1 Standard Deviation of the mean



About 99. % of data is within 3 Standard Deviations of the mean



- A data point's z-score is the # of standard device him away from the mean
- Formula (NOT in AP exam):  $z = \frac{x-\mu}{\sigma}$
- Z-scores can help us compare two unalike measurements

Example: Suppose the weights of apples are normally distributed with a mean of 85 grams and a standard deviation of 8 grams. The weights of oranges are also normally distributed with a mean of 131 grams and a standard deviation of 20 grams. Amy has an apple that weighs 90 grams and an orange that weighs 155 grams.

1. Calculate and interpret the z-score of Amy's apple
$$Z_{\alpha} = \frac{90.85}{8} = .625$$

2. Which is relatively larger, Amy's apple or her orange? Explain.

$$\frac{7}{20} = \frac{155 - 131}{20} = \frac{24}{20} = 1.2$$

The drange, it is more stal der above the mean makes it large-

3. How large would Amy's apple have to be in order to be comparable to her orange?

$$1.2 = \frac{X - \delta S}{8}$$
 $9.6 = x - 85$ 
 $X = 94.69$ 

4. Know how to use Z-scores to calculate the percentage of data points above, below, or between certain boundaries \*This works ONLY for normally-distributed data!! DO NOT do these procedures if you do not know that your data is normally distributed!

#### With Z-table

- Z-table gives the percentage of values below a given z-score
- You can use the z-table backwards if you know the percentage, find it on the z-table, and see what z-score it equates to!

#### With Calculator

13.5% 34%

34% 13.5%

- NormalCDF (if *looking for* percentage/probability)
- InvNorm (if given percentage or probability)
- To adequately *show work*, you must write...

## Topic 5: Probability Rules (Notes: Chapter 6)

## 1. Understand what probability is

How do you calculate the probability of an outcome?
$$P(s) = \frac{\# \circ f \circ s}{5}$$

What is the Law of Large Numbers?

As the number of chance experiments increase the difference by the true value and the relative frequency of success approaches zero What are mutually exclusive outcomes?

Two events that cannot occur similareausty sinvitareausty or two events with no common outcomes

What are *independent* events?

Two events in which the occurance of one event does not effect the occurance of the 2rd event.

Why can two events that are mutually exclusive never be independent? If one of the my trally exclusive events occur then we know the other will not occur so they can not

be independent (the occurance of one changed likely hard of other)

- 2. Know the basic rules of probability
  - When calculating the probability of getting more than one outcome for a given event, what formula should you use? HINT: Always account for any overlap between outcomes!

When analyzing events with multiple outcomes, what visual aide will be the most beneficial?

Venn diagram, tree diagram, two way table

- When calculating the probability of multiple events, what rule or formula should you use?
- When, and how, do you use the combinations (nCr) function in your calculator?

Use when you need a number of arrangements

When analyzing a series of multiple events, each with multiple possible outcomes, what visual aide will be helpful?

When calculating the probability of multiple independent events, what three things should you account

- How does the above procedure change when the events are dependent?
- What is conditional probability, and how do you calculate the conditional probability of a given event?

A probability that is dependent on another event occurring
$$P(A|B) = P(A \cap B)$$

$$P(B)$$

Situation	Rule	Formula
"At least one"	Opposite of "none"	1 - P(0)
Multiple outcomes – mutually exclusive	Add probabilities	$P(A \cup B) = P(A) + P(B)$ $NOTE: P(A \cap B) = 0 $ (no overlap for mutually exclusive events)
Multiple outcomes – NOT mutually exclusive	Add probabilities but <i>subtract the overlap</i> *If using a Venn Diagram, just add up the 3 sections in the diagram	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Multiple events — Independent	Multiply probabilities, and account for COMBINATIONS in which these events can occur (nCr)	$P(X = k) = \binom{n}{k} p^{k} (1 - p)^{n - k}$ $\text{nCr} \bullet (p_{\text{success}})^{\# \text{ of successes}} \bullet (p_{\text{fail}})^{\# \text{ of fails}}$
Multiple events – Dependent	Multiply probabilities  *Account for the <b>change</b> in probability with each trial  *Account for <b>combinations</b> (nCr)	nCr • pevent 1 • pevent 2 • pevent 3  NOTE: Remember these probabilities CHANGE!!
Conditional Probability (A <i>given</i> B)	Probability of both events Probability of first event	$P(A B) = \frac{P(A \cap B)}{P(B)}$

## Topic 6: Probability Distributions (Notes: Chapter 7)

- 1. Know the different types of random variables and how their distributions work
  - What is the difference between a discrete and a continuous random variable?

    A discrete random variable has vilves that are is a leted
    points on a number line.

• For continuous random variables, what is the probability of getting exactly one given outcome?

• How do you calculate the expected value of a discrete random variable?

• What is the **definition** of expected value? (It mean something very specific)

• What formula can you use to calculate the spread (st. dev.) of a discrete random variable by hand?

• How are variance and standard deviation related?

2. Know how transforming and combining a random variable changes that variable's distribution

Action	Effect on Center (mean)	Effect on Spread (standard
A 11' (G 1		deviation)
Adding/Subtracting a CONSTANT	T+ 1 = X + C	
(number)	My= Wx+C	04 = 0x
Multiplying/Dividing by a	If Y=aX	0.7
CONSTANT (number)	Uv=all	0 = lat 0 = 2 0 x
Combining (adding or subtracting two random variables to each	If Z = X+Y	If 2=X+Y
other)	MZ=MX+MY	A2 A2 22
	Wf - MX + MY	102 = 0x +0x
		3 3
		05-10x +QL

## HINTS:

- If X and Y are normally distributed, so are X + Y and X Y. This means use normalCDF!
- X > Y is the same as X Y > 0 (likewise, X < Y is the same as X Y < 0)

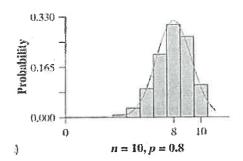
## 1. Know and understand how to use a Binomial Distribution

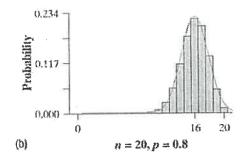
## Using the Binomial Distributions

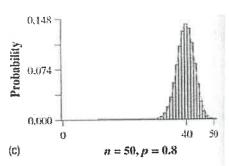
- Only works in *binomial* settings, which occurs when the following conditions are met ("BINS")
  - mutually exclusive outcomes
- BinomPDF: finds
- BinomCDF: finds \( \bigve{Y} \)

#### **Binomial Curve**

- (number of trials probability of success = expected # of successes)
- SPREAD: Standard Deviation,  $\sigma = \sqrt{n} \pi (1-H)$
- SHAPE: Approaches **normality** if you can <u>expect</u> at least <u>successes</u> and







## Example:

Genetics says that children receive genes from each of their parents independently. Each child of a particular set of parents has probability a probability of 0.25 of having Type O blood. Suppose these parents have 6 children. Let X = the number of children with Type O blood.

a. Calculate the mean and standard deviation of the number of children who will have Type O blood

$$\sigma_{x} = \sqrt{6(25)(.75)} = 1.06$$

b. Find the probability of each of the following

P(X = 4); exactly 4 children |  $P(X \le 3)$ ; 3 or fewer

(b)  $(25)(75)^2$ b)  $(4)(25)(75)^2$ b)  $(4)(25)(75)^2$ b)  $(4)(5)(25)(75)^2$ b)  $(4)(6)(25)(75)^2$ 

P(X > 1); More than 1 child will have Type O blood

(12x)4-1c (1cx)

 $P(X \ge 3)$ ; 3 or more children will have Type O

2. Know a	nd ir	nderstand	how to	0 116 <del>0</del> 9	Geometric	distribution
Z. IXIIOW a	mu u.	uuustanu	HOW I	J use a	ствоинение	COSTEADILLION

Geometric Distribution – a density curve that allows us to determine how many trials it will take to get

one success

(also think of it as

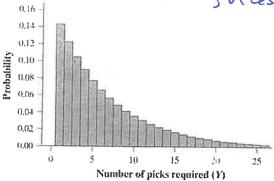
o Events need to be <u>Independent</u> (of course)

Wait time

How to calculate it

Calculator

- GeometPDF is used for growth, the probability that the first success will happen Oh the Kth trial
- GeometCDF is used for the gradifies, the probability that the first success will happen on or before the Kth trial
- Type in probability and the fiel #
- EXPECTED VALUE (mean) of a Geometric Random Variable is \_\_\_\_\_ (If  $n = \frac{1}{v}$ , then np = 1)
- Shape is always Vight
  - O As you continue, the probability of having your first gets lower



## **Examples:**

- 1. A slot machine has a win rate of 8%. A gambler wants to play at this slot machine until they win then, they will leave.
- a. What is the expected number of games the gambler will have to play in order to win?
- b. Find the probability that it will take the gambler...

7 spins to win

10 or fewer spins 
$$p(X \le 10) = g cd + (000, 10)$$
 More than 20 spins  $p(X > 20) = |-p(X \le 20)$ 

More than 20 spins 
$$(X \le 20)$$

## 1. Know the basics of sampling distributions

• What is the difference between a parameter and a statistic?

A parameter is a measure next taken from a population

A statistic is a measure next taken from a sample

What is the difference between a proportion and a mean?

Copartium is the cather of the # that the categorical variable appears

divided by # sample site.

mean is the sum of att the numerical variable for any object in

what is a sampling distribution? the sample divided by the sample site

A collection of all possible sample states to talva from all the possible samples in a population

- Know the difference between a sample distribution and a sampling distribution
  - o Sample distribution a graph of data taken from one sample
  - o Sampling distribution a graph of statistics taken from multiple samples

2. Know the importance of the Central Limit Theorem (define it below)

When n is sufficiently large the sampling dost of x

is well approximated by a normal curve, even when the

population is not itself normal

- 3. Know how to analyze a **normal distribution**, and use it to find the probability of a sample statistic occurring, *given* an assumed population mean and standard deviation

#### From the AP Formula Sheet:

If X has a binomial distribution with parameters n and p, then...

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If  $\overline{x}$  is the mean of a random sample of size n from an infinite population with mean  $\mu$  and standard deviation  $\sigma$ , then...

$$\mu_{\overline{x}} = \mu$$

$$\sigma_{\overline{\chi}} = \frac{\sigma}{\sqrt{n}}$$

• **REMEMBER:** These formulas are for **CONVERSION** from the population standard deviation! If you're already given the standard deviation of the sampling distribution, just use it!

4. Know the **CONDITIONS** that must be met for the Central Limit Theorem to apply, and thus for **inference** to occur Condition How to meet the condition Shape of the sampling distribution is appropriate for inference (center, shape, or spread) For Proportions: NTT 210 and n (1-11) 214 For Means: NOTE: If the population has an Parent Pop :5 normal approximately normal distribution, this condition can be considered "met" graphical display regardless of sample size! 2. were randon / assigned

What must we do if the conditions are not met?

PROCEDURES FOR CONFIDENCE INTERVALS AND SIGNIFICANCE TESTS (Chapter 9-11)

## 1. State what you're doing

Confidence Intervals

- Procedure you're using
- The *parameter* (population) *of interest!*
- Confidence level

"We will use a \_\_\_\_\_ Interval to estimate, with \_\_\_\_ % confidence, the *true* (mean/proportion) of \_\_\_\_\_ (context)

## Significance Tests

- Procedure you're using
- The parameter (population) of interest!
- Hypotheses, H<sub>0</sub> and H<sub>a</sub>
- Significance Level, α (If none is given, use .05)

"We will use a \_\_\_\_\_ Test to test the following hypotheses at the  $\alpha$  = \_\_\_\_ level"

#### Additional Notes:

- Remember, H<sub>0</sub> implies "no change" or "no difference"
- If you are doing a 2-Sample or 2-Proportion test, state **both** populations indicate which one is which!
- For a **Paired** t-test, find the *difference* between the matched pairs, and use these *differences* as your one sample!  $\mathbf{H_0}$ :  $\mu_{\text{Difference}} = 0$ ,  $\mathbf{H_a}$ :  $\mu_{\text{Difference}}$  is >, <, or  $\neq 0$

## 2. Check your conditions

<u>NOTE:</u> If a problem says "assume conditions are met", you do not have to go through this process!!

- Sample Size (also known as "Large Counts")
  - $\circ$  If met, the SHAPE of the <u>sampling distribution</u> is Normal (or  $\chi^2$  distribution for  $\chi^2$  tests)
  - Means (μ):
    - 30 or more, OR
    - Graph of the sample shows no obvious skews or outliers (t-test only), OR
    - Population is *known* to be normal
  - o Proportions (p):
    - At least 10 expected successes and 10 expected failures (find expected value of each)

a	Ran	do	mn	1000
•	TX A.H	(11)		1000

- Ensures that the CENTER (the <u>sample statistic</u>) is legitimate
- o Samples and Observational Studies: Randomly selected from the population
- Experiments: Randomly assigned into treatment or control group(s)
- Note: If you are running a 2-sample interval or test, you must <u>check</u> and <u>STATE</u> that <u>both</u> samples are random!
- Independence

Confidence Intervals

Give interval: (lower, upper)

- o Ensures that the SPREAD (the standard deviation) formulas that you're given are reliable
- o Samples and Observational Studies: sample must be less than 10% of the population
- o Experiments: Groups should be independent of each other (i.e. not matched pairs)
  - If there ARE matched pairs, do a PAIRED t-test; find the *difference* between each pair and use *those* numbers in a 1-sample t-test!

Significance Tests

Test Statistic (z, t, or  $\chi^2$ )

Degrees of Freedom (t and  $\chi^2$  ONLY)

## 3. Do the calculation (create the interval or run the test)

Re-state *type* and *confidence level* (just to be safe)

	• p-value
4. State your conclusion	
Confidence Intervals	Significance Tests
• Give the % confidence	• State whether $p < \alpha$ (reject) or $p > \alpha$ (fail to
• Give the interval in context (including PROPER	reject)
UNITS)	Give the consequences in context
	Chi-Squared: You may be asked to perform a
"I am% confident that the true mean (or	follow-up analysis to see where the biggest gaps
true proportion) of <u>(context)</u> is between	between observed and expected values are.
and"	
	REJECT: "Because $p < \alpha$ , we can reject $H_0$ . There
The true mean value is between	is statistically significant evidence to suggest
The true mean value is between (low) and (up) and - 1. ot all	(whatever H <sub>a</sub> was)
I had same way will	FAIL TO REJECT: "Because $p > \alpha$ , we fail to
intervals created the same way will contain the true mean	reject H <sub>0</sub> . There is NO statistically significant
Contain the true week	evidence to suggest (whatever $H_a$
CONTEXT!!!	was)

IMPORTANT: The p-value is ALWAYS between \_\_\_\_\_\_ and \_\_\_\_\_. If you calculator gives something other than this, I <u>guarantee</u> there will be an E at the end. This represents <u>scientific notation</u> (# • 10<sup>x</sup>). This means your p-value is very small (in fact, many statisticians just write "p < .001" and call it a day). As far as we're concerned, p-values this low will <u>always</u> be significant!

ALSO IMPORTANT: Know the difference between "interpret the p-value" and "draw conclusions"

- Interpretation: IF H<sub>0</sub> is true, the probability that we would get a test statistic as or more extreme as the one we got in our sample (by random chance) is \_\_(p-value)\_\_
  - o **NOTE:** If there is a *direction* involved (< or >), <u>state</u> that direction ("as high or higher" or "as low or lower")

• Draw conclusions: Rejecting or Failing to Reject H<sub>0</sub> (and associated context)

**Topic 9:** Confidence Intervals (Notes: 9.1,9.2,9.3,11.1-11.3)

Understand the purpose of confidence intervals and how they work

• What does a confidence interval allow us to do?

Estimate a population parameter with a Certain degree of confidence

• How do we *interpret* a confidence interval? (For instance, to interpret 95% confidence level, what *words* would you say?)

The true parameter is between \_ and \_, and \_ \_\_\_, and \_\_\_\_, and \_\_\_\_, of all intervals created the same way will contain the true parameter

How do we interpret a confidence level? (For instance, in a 95% confidence interval, what does the 95%

- tell us? What does it mean to be "95% confident"?)

  It tells us the percent of time, in the long run, that
  the true parameter falls inside the interval created
- Know how to use the FORMULA for confidence interval:
  - O Statistic ± Critical Value Standard Deviation of Statistic
  - o Critical Values can be found in the t table (for z distributions, use the \_\_\_\_\_ row)
  - O Standard Deviation: Use the formula sheet (they are very clearly laid out!)
    - In this context, St. Dev. of the Sampling Distribution is also called Standard Error
- What is the margin of error, and how do we calculate it?

the difference by the sample statistic and the lower or upper bond of the interval.

ME = (Critical value (Std error)

2. Know what type of confidence interval to calculate, and when to calculate it

When estimating a population proportion

When estimating the *difference* between two population proportions

When estimating a **population mean** and the population standard deviation is *known* (RARE)

When estimating the *difference* between two population means and the population standard deviations are *known* (RARE)

$$\bar{X}_1 - \bar{X}_2 \pm 2 \times \int_{\Omega_1}^{\Omega_2} + \frac{\sigma_1^2}{\Omega_2}$$

Note: The true name of this procedure and the calculator name are slightly different. Know both!

When estimating a **population mean** and the population standard deviation is **NOT** known

Note: The true name of this procedure and the calculator name are slightly different. Know both!

When estimating the *difference* between two population means and the population standard deviations are **NOT** known

$$\bar{X}_1 - \bar{X}_2 \pm \pm \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

## 3. Know the essentials of the *t*-distribution

• When do we use it?

• How do we calculate the degrees of freedom of a t-distribution?

## 4. Know the general process of statistical inference (in this case, creating a confidence interval)

- 1. Check conditions
- 2. Calculate critical value
- 3. construct interval
- 4. Conclusion

#### 5. Know how to check conditions

• What conditions must you check, and where in the study guide can you look to find them?

• If dealing with a *t*-distribution and your sample size is not 30 or more, what *other* methods can you use to check for normality? **Be specific!** 

6. Know how to manipulate confidence intervals
• Be able to solve for $n$ or $z^*$ (or $t^*$ ) (NOTE: In multiple choice, you can always plug in the choices!)
o If a sample proportion is not given in this case, assume $p =$ (this gives us the greatest margin of error to work with)
• Remember that the sample statistic ("point estimate") is in the of the confidence
interval (and that the distance between the sample statistic and the ends of the confidence interval is the
margin of error
• Know what happens to the margin of error (and thus width of the confidence interval) if we
o Increase sample size:
o Decrease sample size:
o Încrease confidence level:
o Decrease confidence level:
• If you adjust sample size, confidence interval changes by the square root of that amount (since n is inside the square root in all standard deviation formulas)  • Example: What will happen to the confidence interval if you multiply the sample size by 4?  The interval width will be \frac{1}{2} as wide
Topic 10: Significance Tests (Notes: Chapters 10 & 11)
1. Understand what significance tests are for and allow us to do
• What are the two types of hypotheses used in significance tests, and what symbols do we use to represent them?
Alternature - Ha
• What is a null hypothesis, and what does the null hypothesis always assume to be true?  NOTE: The answer is slightly different for 1-sample and 2-sample tests - know both!  The null is what we assume to be true. We assume that  the parameter of interest is equal to the hypothesized value
• What is an alternative hypothesis? What are the 3 types of alternative hypotheses you could have?  NOTE: The answer is slightly different for 1-sample and 2-sample tests – know both!  A competing claim.

- 2. Know what type of significance test to run, and when to run it

2. Know what type of significance test to run, and when	a to tun it
When testing a claim about a population proportion	When testing a claim about the difference between two
Large sample & test of proportion	population proportions Zindependent sample 2 test, deference
	of propertion
When testing a claim about a population mean and the	When testing a claim about the difference between two
population standard deviation is <u>known</u> (RARE)	<b>population means</b> and the population standard deviations
Large sample & test of mean	are known (RARE)
Large sample & test of the	2 independent sample 2 test, differen
	of means
When testing a claim about a population mean and the	When testing a claim about the difference between two
population standard deviation is NOT known	<b>population means</b> and the population standard deviations
Large sample t test of mean	2 independent sample t test, differe
	of means
When testing a claim about a study or experiment that	In the calculator, which type of test would you select?
utilizes matched pairs 2 dependent sample thest	I sample & fest, usual the
différence of means	1111 4 , or with the

- 3. Know how to *interpret* the results of a significance test
  - What two (for t-tests, three) things should you report after running a significance test in your calculator?

test state, p-value, df

- How do you interpret a p-value? What does that number mean?

  If p-value is < MOX then the Ho is rejected

  P-value is the probability of getting a sample this rare or more

  rare assuming the null is true
- How do you analyze (interpret the results of) a test for which the p-value is *less* than alpha (for instance, p < .05). What would you write?

Reject the Ho, There is sufficient evidence to support the class that Alternative Hyp.

• How do you analyze (interpret the results of) a test for which the p-value is *greater* than alpha (for instance, p > .05). What would you write?

Fail to Reject the Ho, There is not sufficient evidence to support the alternative hyp.

## Topic 11: Chi-Squared Tests and Types of Error (Notes: Chapter 12)

1. Know the similarities and differences between Chi-squared and the other types of significance tests

• When do we use Chi-squared tests? In other words, what do Chi-squared tests allow us to measure?

When analyzing the distrota categorical variable.

• What are the three types of chi-squared tests, and when do we use each?

Туре	Purpose/When to use	Name in Calculator	
K2 GOF test	One sample, comparing the dist. of a categorical variable to an expected dist.	12-60F	
the fest of Humogeniuty	two samples or more, Comparing the dishibution of ai categorical variable to two or more populations	x2-Test	
K² fest of Independence	One sample, two categorical variables, checking to see if relationship or association blue the two variables	K'-Test	

**NOTE:** The biggest difference between the second and third type is <u>context</u>. Other than that, they are essentially the same.

What are the null and alternative hypotheses of a Chi-squared test?

Ho! The distribution is as expected
Ha! The distribution is not as expected

## 2. Know the conditions of a Chi-Squared test

- Same conditions as other significance tests
- How is the sample size condition different for Chi-Squared tests, and how do we check it?

SRS + Large Enough: all expected cells ≥ 5 Independence of samples and subjects

- 3. Know how to calculate and interpret the Chi-squared statistic
  - How can we find expected counts?
    - o Goodness-of-fit: **READ THE PROBLEM!** 
      - Sometimes, you may expect certain proportions out of a total (like we did with M&Ms).
      - Sometimes, you may expect that the data is equally distributed among the categories (in this case, just use simple division!)
    - Homogeneity and Independence: What formula do we use to calculate each expected value?

- How do we calculate *degrees of freedom* for a chi-squared test?
  - o Goodness-of-Fit: # cat 1
  - o Homogeneity and Independence: ( ~ w ~ 1 ) ( c ( 1 )
- When running a Chi-Squared test, what three things must you report? NOTE: The interpretation and analysis/drawing conclusions aspects of these are the same as the other significance tests.

4. Know what Type I and Type II error are; be able to spot them in context, and discuss what the consequences of these types of error would be if they happened in a real-life situation (including possibly evaluating which one would be worse in that situation) (Notes for 10.2)

HINT: The chart on your 5.4 notes may be a handy tool to help you understand and remember which is which!

What is a Type I error?

What is a Type II error?

What variables are used to represent the probability that Type I error and Type II error, respectively, will happen? P(type I) = d

- 5. Know what power is, why it's important, and how it can be influenced. (Activity for 10.5)
  - What is the definition of *power*?

How is power calculated?

How can power be *increased*? List 3 ways.

- 1) Increase sample size tresearcher control

  (3) Increase d (significance level) researcher control

  (3) Increase distance from hyp. value to actual value to micant be

  controlled
- 6. Understand the relationship between Power, Type I Error, and Type II error

Power	Type I Error (α)	Type II Error (β)
Increases 1	1	11
Decreases J	4	1

Fill in each of the following blanks with either "same" or "opposite"

Type I and Type II error always go the Opposite

Power and Type I error always go the \_\_\_\_\_\_ Same\_\_\_\_\_ direction

Power and Type II error always go the Oppos: Le

Suppose you want to avoid a Type I error at all costs. Should you use a significance level of .10, .05, or .01? Explain.

## **Topic 11:** Bivariate Data (Notes: Chapter 5)

- 1. Know how to analyze a correlation between two variables
  - Explanatory and Response variables (which one is x and which one is y?)

x is the explanatory
y is the response

• 5 things we should look for in bivariate data:

Characteristic	Possibilities	What the <b>r-value</b> tells us
Shape	are the ordered pairs have a linear shape	R-value assumes that shape is
Strength	are the points close to the LSRL	r is close to
Direction	from left to right or are they dec. ??	+ if inc - if dec

Outliers (especially if they substantially alters the equation of the regression line, or line of best fit)

Context (as always) – what two variables are we examining?

· X and Y are correlated. Does this mean that X causes Y?

No correlation closs nut imply causation

2. Know how to analyze the least-squares regression line (line of best fit):  $\hat{y} = mx + b$ 

- ŷ is the predicted value of y for a given value of x
- · Interpretation of Slope:
  The amount we would expect, on average, change in y if it every

  lunit increase in x -> remember to add contex + units
- Interpretation of Y-intercept:
- r² value ("coefficient of determination")

  The percent of variation in y that is due to the linear relationship both x and y. -> remember context
- Extrapolation only should predict values for x's that are with the domain used to create the LSRL

... 3. Know how to analyze residuals and residual plot

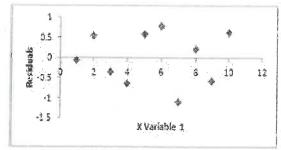


What is a residual?

The diff. by the actual and predicted valves

Resid = y - y

How do you calculate a residual?

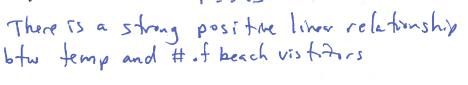


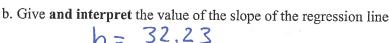
What information does a residual plot give you?

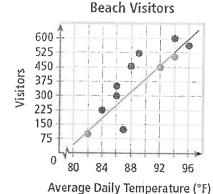
If there is a relationship by X's and residuals

- 4. Know how to handle *curved* data (linear transformations)
  - Be aware that one or both variables may be transformed (square root, log, natural log (ln), etc.) in order to linearize curved data
  - Make sure that all interpretations (see above) take all transformations into account!

## Example







We would	In # afvisifors to beach for	errin
in crease	in # afvisifors to beach for	eval
1 degree	incrase in temp.	

		-		
Predictor	Coef	SE Coef	t	P
Constant	-2486.13	96.84	-2.11	.03
Temp	32.23	15.3	4.76	.000
r = .85		$r^2 = .72$		

c. Give and interpret the value of the y-intercept of the regression line

d. Give and interpret the r<sup>2</sup> value of the regression line

72% of the variation in the number of visitors is due to the linear relationship by # of visitors and temperature

e. If tomorrow's temperature is going to be 90°, predict how many visitors the beach will have tomorrow. Show work!

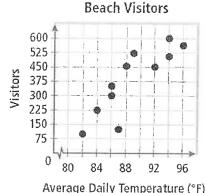
	Topic 13: Confidence Intervals and Significance Tests with Bivariate Data (Notes: Chapter 13)
•	A regression line is created using a bivariate set of data. Confidence intervals and significance
	tests allow us to predict and test the amount slope of the relationship between the explanatory
	and response variables (x and y)
	o You can also do this for y-intercept, but this is not something to worry about for the exam
Đ	The AP exam will most likely ask you to use a Computer out put to make inference
	o Remember, everything dealing with slope is in the row with the variable name
	("constant" refers to the y-intercept)
	o If you need to do them in the calculator
	■ 1. Put all Xs in one list and Ys in another list
	• 2. Go to LinRegInterval or LinRegT-Test, type in the inputs, and get your results!
•	Confidence Interval
	o Confidence interval = Statistic ± Critical Value • Standard Deviation of Statistic
	o For a linear regression, this becomes b ± £ * 56
	"SE Coef' can be found in the Computer output col. 3
	t* can be found in yourcalculator
	<ul> <li>For degrees of freedom (DF), use</li></ul>
	o Interpretation (assuming 95% confidence)
	I am 95% confident that the slope of the true regression line of the relationship
	between <u>x</u> and <u>y</u> is between <u>and</u> .
•	Significance Test
	o Ho: Assume that there is <u>No relationship</u> between the variables (this means
	slope $(\beta) = \underline{\mathcal{O}}$
	o Ha can be,, or (just like before)
	o t and p can both be found in the $row$ of $slope$ . Interpret as usual!  The formula for the test statistic is
	The formula for the test statistic is

- Conditions!! (Use the acronym LINEaR)
  - o L: The dist. of e at any x is zero. That is Me=0
  - o I: The std. dev of e at (or use X is the same,
  - o N: The dist of e at any x is approx. normal
  - o E: the e's assoscrated with diff observations are independent
  - o and
  - o R: andon sample

## Example

The Florida Tourism Department is studying the habits of beachgoers across the state. They observe a certain beach on 11 randomly-selected days during the peak season (May thru August) and record the Average Daily Temperature and the number of visitors who come to the beach that day. A scatterplot of the data is shown, as is a computer output of the data.

Assume that temperatures and number of visitors are both normally distributed.



		HVCIQ	ge Daily	iembergrate (
Predictor	Coef	SE Coef	T	Р
Constant	-2486.13	96.84	-2.11	.03
Temp	32.23	6.76	4.76	.000
r = .85		$r^2 = .72$		

a. CHECK conditions for inference:

A linear model seems appropriate so all 4 basic assumptions are met.

The sample was randomly selected

b. Construct and interpret a 95% confidence interval of the slope of this regression line

$$t^{*} = invt(.025, 9) = 2.262$$
 $b + t^{*}S_{h}$ 
 $t = 32.23 + 2.262(6.76)$ 
 $t = 16.94, 47.52$ 

The true slope of the relationship by the free slope of the relationship by the free slope and temp is by 16.94 pp/der and 47.52 pp/der. And 95% of all intervals created the same way will contain the true slope.

c. Is there significant evidence at the  $\alpha = .05$  level to suggest that there is a relationship between average daily temperature and number of visitors?

Yes, p-value for slope is approx. Zero, therefore we would reject the null inflavor of there being a relationship between # of visitors and temperature

#### **Additional Topics**

1. Know the **SYMBOLS** for parameters and statistics. **Mis-using a symbol** <u>WILL</u> cause you to get docked on the exam!!

Measure	Parameter Symbol (Population)	Statistic Symbol (Sample)	
Mean	μ	$\bar{x}$	
Standard Deviation (also applies to Variance)	σ	S	
Proportion	p or $\pi$	$\hat{p}$ or <b>p</b>	
Sample size	n		

- 2. Know how to work with percentiles ("relative frequency")
  - A data point's percentile tells the percentage of the data that is less than or equal to that data point
    - o **Example:** If you're in the 85<sup>th</sup> percentile, 85% of the population is <u>at or below</u> your level
    - O This means Q1 is the 25<sup>th</sup> percentile, Median is the 50<sup>th</sup>, and Q3 is the 75<sup>th</sup>
  - The numbers in the z-table can be considered *percentiles* (for instance, the z-score 0.45 corresponds with .6736 in the z-table, which is the 67<sup>th</sup> percentile)

#### AP EXAM ADVICE

General advice for ALL your exams:

- Be prepared
  - o Have your pencils and materials ready to go
  - o Get a good night's sleep! (This will feel strange to some of you)
  - o Be on time. You WILL NOT be admitted to the testing room if you are late.
  - o Leave the personal drama at the door. Do not let it bring you down on an exam this important!
- Don't try and do too much! I have seen many students write great answers, only to get docked because they added an incorrect piece of information or tried to make a claim that wasn't there. Answer the question as fully yet concisely as possible, and then get out!
- Read each question **VERY** carefully! AP loves to throw curveballs and you need to be sure of what the question is asking you to do!
- TIME IS OF THE ESSENCE. If you are stuck on a question, **OR** you know that question may take a while to figure out, *come back to it*. Knock out the easier ones first.
- Two minute warning is the best time to start guessing (especially on Multiple Choice).
  - o The WORST answer you can possibly have is a blank!

- Specific advice for THIS exam:
  - TIMING:
    - o 2 minutes and 15 seconds for each multiple choice
    - o 13 minutes for Free Response #1 5
    - 25 minutes for Free Response #6
    - Some questions will take more or less than this. That's fine. Just pace vourself!
  - Calculator Check!
    - o Is it charged and/or have working batteries?
      - If your TI-84 is okay at the start of the test but then says "low battery" in the middle of the test, it will last through the duration of the test. DO NOT WORRY!
  - Show work! You HAVE to show enough to prove to the AP Readers that you understand the process behind your
    answers (you WILL get docked for not showing enough work.)
    - o It doesn't matter how simple the calculation is. If it's 1+1=2, write that down.
  - **Formula sheet** is your friend! *Especially* the 2<sup>nd</sup> and 3<sup>rd</sup> pages (as well as the **t-table** because it gives you all the *critical values* you could ever want!). Sometimes the formula sheet gives away an otherwise tricky answer.
    - But be careful: do not, and I repeat, **DO NOT TEAR OUT THE FORMULA SHEET FROM THE**TEST BOOKLET. THIS WILL INVALIDATE YOUR EXAM. This happened to someone I knew on the AP Chem Exam; her score was invalidated and she had to take the test again next year.
  - If you need to make a graph, **LABEL YOUR AXES!!** 
    - If you're doing it to check the Normality (Sample Size) condition for inference, make sure you write whether you see any skews or outliers. **Just showing the graph is not enough** (but don't *forget* to put the graph, either! You need BOTH the graph AND the analysis of skew/outliers)
      - Remember that boxplots are the most efficient (but not the only) way of checking for this!
  - Watch your language! Words like average, range, skew, and significant have very specific meanings in statistics, so DO NOT use these words unless you are using them in the correct statistical context (otherwise, find synonyms)
    - Average → Typical
    - o "Ranges from" → "Goes from"
    - o Skews → Distorts
    - o Significant → Substantial
    - NOTE: It is okay to use these words for their <u>statistical</u> definitions. Just use synonyms if you're going to venture outside of that.
    - o If you aren't sure what a word means, avoid using it!!
  - Stick to the script! Know how to phrase your analyses of the following (they are in your study guide). These phrasings help ensure you have covered all important aspects of the analysis in a clear and concise manner!
    - Confidence intervals
    - o Confidence levels
    - o Interpreting p-values
    - Analyzing or drawing conclusions about p-values
    - o Slope of a regression line
    - o Interpreting r<sup>2</sup>

- Randomization and a large sample size can solve most of life's problems they make for better, more accurate,
   and more reliable (unbiased) results
- DO NOT mix up the language of sampling and the language of experiments.
  - For example, subjects of experiments are usually not randomly selected (often times that's *highly* unethical). They *are*, however, randomly *assigned* to groups (at least they *should* be)
- If you use symbols, **DEFINE** what that symbol means. OR you can weave the context *into* your symbol o *Both ways are acceptable* (although one is definitely **quicker!**)

Symbols with definitions	Symbols with context interwoven
$P(A \cap B)$ , where A represents being a girl and B represents being a senior	P(Girl ∩ Senior)
$\mu$ = 23, where $\mu$ represents the mean weight of the population of piglets (or true mean weight of piglets)	$\mu_{\text{piglets}} = 23$
$p_1 > p_2$ , where $p_1$ represents the <i>true</i> proportion of adults who like snacks, and $p_2$ represents the <i>true</i> proportion of children who like snacks	$p_{ m adults} > p_{ m children}$

- For sampling distributions, make sure you use  $\mu_{\bar{x}}$  (or  $\mu_{\hat{p}}$ ) for mean and  $\sigma_{\bar{x}}$  (or  $\sigma_{\hat{p}}$ ) for standard deviation
  - o <u>IF YOU DON'T KNOW WHAT SYMBOL TO USE</u>, <u>DON'T USE A SYMBOL AT ALL!!</u> There's nothing wrong with writing out an answer in words. An incorrect symbol WILL get you docked.
- For inference problems (confidence intervals and significance test), LOOK for the statement "assume all conditions are met". If it is not there, you had better check those conditions!
  - Also be on the eye out for *randomness* is it stated? And for 2-sample problems, is it stated for *both* samples?
- If you're doing an interval or test, always provide the name of the procedure when you do it!
- Remember, **NEVER** claim H<sub>0</sub> or H<sub>a</sub> are "true" or "false". We "reject" or "fail to reject" based on the *probability* of getting a certain result by chance (that's what significance tests are all about!) and we *know* that probability is NEVER a guarantee!
- **BREATHE!!** We've been working for this all year. *You've got this!* One wrong answer won't kill you. Heck, just getting half of the questions right is *almost guaranteed* to be a 3! Don't overthink just do your best.

